Monetary Policy when the Phillips Curve is Quite Flat

Paul Beaudry, Chenyu Hou and Franck Portier*

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Abstract

This paper highlights how the presence of a cost channel of monetary policy can offer new insights into the relation between monetary policy and inflation when the Phillips curve is quite flat. For instance, we highlight a key condition whereby lax monetary policy can push the economy in a low inflation trap and we discuss how, under the same condition, standard policy rules for targeting inflation may need to be modified. In the empirical part of the paper we explore the relevance of the conditions that give rise to these observations. To this end, we present both *(i)* a wide set of estimates derived from single-equation estimation of the US Phillips curve and *(ii)* estimates based on structural estimation of a full model. The results from both sets of empirical exercises strongly support the key condition we emphasize.

Keywords: Monetary Policy, Inflation, Interest Rates.
JEL Code: E3, E32, E24.

Introduction

Prior to the Covid-crisis, the inflation rate has been below target for several years in many industrialized countries. At the same time, monetary policy has being sufficiently expansive to support unemployment rates that were close to historical lows. These low inflation outcomes could have reflected a correlated reduction in the natural rate of unemployment across countries. However, such explanation appears unlikely given that only a few years ago the predominant puzzle was missing deflation with high unemployment. A more plausible candidate explanation for these outcomes is that the Phillips curve may be quite flat.[1]

*Beaudry: Bank of Canada/Banque du Canada, Hou: Chinese University of Hong Kong, Shenzhen, Portier (corresponding author): University College London and CEPR. The authors thank Edouard Challe, Behzad Diba, Davide de Bortoli, Hashmat Khan, Olivier Loisel, Xavier Ragot and participants to the seminars where this paper's ideas were presented. Some of the ideas and results in this paper were first circulated under the title" Real Keynesian Models and Sticky Prices"(Beaudry and Portier [2018]).

[1] By "slope of the Phillips curve", we mean the partial relationship between inflation and a measure of market tightness such as either the output gap or the labor gap.

The object of this paper is to explore the implications and empirical relevance of a relatively flat Phillips curve when a cost channel is present. The paper is divided into two main parts. In a first section, we highlight a set of theoretical implications of having a flat Phillips curve in the presence of a cost channel. As we shall show, this type of environment will offer a simple explanation for why inflation can get stuck below target, with low unemployment even if monetary policy appears quite aggressive. Such an outcome depends on parameters of the Phillips curve as well as on the sensitivity of aggregate demand to interest rates. Accordingly, in the second and third sections of the paper we explore the empirical plausibility for the parameter configuration of interest. To this end, we present both *(i)* estimates derived from single-equation estimation of the Phillips curve and *(ii)* estimates based on structural estimation of a full model.

In terms of monetary policy, the findings of this paper have novel implications for how policy should be conducted to keep inflation close to an inflation target. In particular, our framework suggests that, when trying to compensate for past departures from inflation target, inflation targeting central banks should not aim for quick redress by adopting slightly more aggressive non-standard interest rate policy. Within our framework, this is precisely the type of strategy that can cause a persistent deviation of inflation from target. In such a situation, it is likely best to leave bygones be bygones and return quickly to a historical rule that has given good inflation results in the past.

The cost channel has been extensively studied in the literature.[2] It was mentioned by Farmer [1984], then modeled by Blinder [1987], Fuerst [1992], Christiano and Eichenbaum [1992], Barth and Ramey [2002] and discussed in the framework of the New Keynesian model by Christiano, Eichenbaum, and Evans [2005], Chowdhury, Hoffmann, and Schabert [2006], Ravenna and Walsh [2006], Rabanal [2007], Ravenna and Walsh [2008], Surico [2008], Tillmanm [2008], Henzel, Hülsewig, Mayer, and Wollmershäuser [2009] and Castelnuovo [2012]. We contribute to this literature by both highlighting the implications of a cost channel when the Phillips curve is quite flat, and by providing two sets of empirical results (single equation estimation of the Phillips curve and multiple equation structural estimation) that support the key parameter configuration that provides novel insight.

The remaining sections of the paper are structured as follows. In Section 1, we derive some simple theoretical implications of having a relatively flat Phillips curve when a cost channel may also be operative. We show under which conditions this can change how an inflation targeting monetary authority should conduct policy to stabilize inflation. In this section we also discuss how an economy can get stuck in a low inflation trap. In Section 2, we begin by examining the plausibility of the parameter configuration of interest by presenting estimates of the Phillips curve when we allow for a cost channel. Given the challenges in estimating the slope of the Phillips curve with aggregate data, we also rely on the slope estimate provided in Hazell, Herreño, Nakamura, and Steinsson [2020] that is identified exploiting U.S. regional variations,[3] so as to focus on the relative strength of the cost channel. In Section 3, we complement this partial equilibrium evidence by presenting

[2] It is worth noting that there are very few studies of the relevance of the cost channel using firm level data. An exception is Gaiotti and Secchi [2006] that find robust evidence in favour of the presence of a cost channel.

[3] See also Fitzgerald and Nicolini [2014] and McLeay and Tenreyro [2020] for the identification of the Phillips curve slope using regional variations.

estimates derived by an estimation of the full model. Finally, in Section 4, we offer some concluding comments.

1 Monetary Policy Implications of a Quite Flat Phillips Curve

The aim of this section is to highlight how the slope of the Phillips curve – or more precisely the sensitivity of the real marginal cost to market tightness – can affect the link between monetary policy and inflation stabilization in the presence of a cost channel. We will first highlight a condition – referred to as a "Patman condition" – under which restrictive monetary policy can potentially increase inflation. We will then examine the implications of this condition for monetary policy.[4]

1.1 The Patman Condition

Since most of the elements of the model we use are rather standard, explicit derivation of the main equations presented here and used in the estimation is presented in C. Our starting point is the two key equations of the canonical New Keynesian setup, where our micro-foundations introduce only minor changes. As is standard, all variables are expressed in deviations from the steady state. We are abstracting from capital accumulation, and we are assuming that technological progress follows a deterministic trend. Deviations of economic activity from its steady state therefore correspond to deviations of employment from its steady state. As a result, when talking about market tightness, we can refer interchangeably to the output gap or the labor gap.

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \, \mathrm{mc}_t + \mu_t, \tag{1}$$

$$y_t = \alpha_y E_t y_{t+1} - \alpha_r (i_t - E_t \pi_{t+1}) + d_t, \qquad 0 < \alpha_y < 1. \tag{2}$$

Equation (1) is the New Keynesian Phillips curve where inflation depends on expected inflation, the real marginal cost and on a markup shock μ_t. This equation is entirely conventional. Equation (2) is a Euler equation (the forward looking IS curve) which is subject to preference shocks d_t. In this equation, we allow for a discounted Euler equation specification[5] by having $0 < \alpha_y < 1$. Such a modification is not very substantial as we allow α_y to be arbitrarily close to one. However, it has the advantage of allowing us to consider a wider set of monetary policy rules without needing to worry about a unit root (induced when α_y is exactly equal to one). In particular, we are able to consider environments where a central bank aims to influence real interest rates which, in addition to being plausible, will be very convenient.

The main element we want to focus on is our specification of the real marginal cost as given by Equation (3):

$$\mathrm{mc}_t = \gamma_y y_t + \gamma_r (i_t - E_t \pi_{t+1}). \tag{3}$$

[4] Throughout this section, we will only be examining positive implications of different interest rate stances with a focus on stances aimed at stabilizing inflation. We will not be doing welfare analysis nor looking for optimal rules.

[5] We provide micro-foundations for this in C.1.1.

In Equation (3), we have the real interest rate included in the marginal cost. In C.2, we show how this formulation can arise in the presence of intermediate goods that are financed at the beginning of the period. It is common to refer to this term γ_r as the cost channel of monetary policy, even though the cost channel of monetary policy is most often associated with nominal interest rates affecting the real marginal cost. As our main results are not substantially modified by allowing for a nominal versus a real interest rate in the cost channel, we choose to maintain a real cost channel specification which is theoretically appealing, offers clearer results and, most importantly, finds greater support in our later estimation.

The central message we want to convey in this section is that the way monetary policy influences inflation is closely tied to a particular condition involving α_r, γ_y and γ_r. We will call the relevant condition the Patman condition after US Senator Patman who argued in the late 1970s that the Fed's policy of increasing interest rates could be more of a contributor to inflation than a cure.[6]

In an economy given by Equations (1), (2) and (3), a marginal increase in the real interest rate $i_t - E_t\pi_{t+1}$ has two effects on current inflation π_t, holding expectations constant. A direct effect γ_r goes through the impact on the marginal cost in the Phillips curve. An indirect effect $-\alpha_r\gamma_y$ runs through y_t *via* the Euler equation. When the direct effect dominates the indirect effect, then an increase in interest rates tends to put upward pressure on inflation. We will refer to such a configuration as satisfying the Temporary Equilibrium (T.E.) Patman condition defined as follows.

Definition 1. Temporary Equilibrium Patman Condition: *Current inflation will increase following a rise in real interest rate, holding expectations constant, if and only if the T.E. Patman condition* $\gamma_r > \alpha_r\gamma_y$ *is satisfied.*

It is important to point out that many models with a cost channel have micro-foundations that rule out the possibility that $\alpha_r\gamma_y \leq \gamma_r$ (see B), that is, they rule out by assumption the possibility of this Patman condition being satisfied.[7] However, this is not the case for the micro-foundations we present in C. Also note that this condition is stated *holding expectations constant*, and for this reason we refer to it as a temporary equilibrium condition. As we shall see below, when we allow inflation expectations to adjust, the T.E. Patman condition will be only a necessary condition for an increase in interest rates to increase inflation. The more complete (general equilibrium) condition will also involve α_y and the persistence of the monetary shock.

[6] The view that tight monetary policy could be inflationary was discussed in Tobin [1980]:

> *"More fundamentally, heretics from the populist Texas Congressman, Wright Patman, to John Kenneth Galbraith have disputed the orthodox view that tight money policies are anti-inflationary, claiming that borrowers mark up interest charges like other cost." (page 35)*

See also Driskill and Sheffrin [1985] who introduced interest costs into Taylor [1979] model of overlapping wage contracts.

[7] However, this is not the case for some larger DSGE models. See for instance Rabanal [2007].

1.2 A First Look at Data through a Tight Lens of the Model

The main idea behind the Patman condition is that monetary policy can have unconventional effects on inflation if increases in interest rates cause marginal cost to increase. In this subsection we want to have a first look at the possibility that monetary tightening may cause marginal cost to rise taking the micro-foundations presented C.2 seriously as a way to model marginal cost. As shown in C.2, the model's marginal cost can be expressed as

$$mc_t = \frac{b}{b-1} \times \text{labour share}_t + \frac{\beta}{b} E_t \left[\frac{1+i_t}{1+\pi_{t+1}} \right], \tag{4}$$

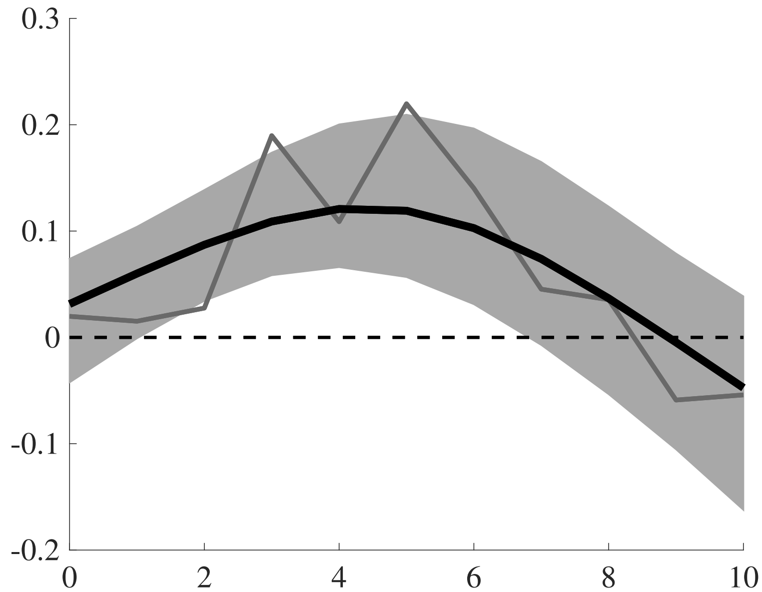
where b is the inverse of the share of intermediate inputs in gross output and β the firms' discount factor. To operationalize this measure of marginal cost, we set $\beta = .99$ and we set $b = 2.28$ based on 2005 data. We measure P_t with the US Domestic Producer Prices Index for Manufacturing and the nominal interest rate as the 3-Month AA Financial Commercial Paper Rate to compute the time series of the real marginal cost implied by the model. Since the resulting series has an important trend, we focus on the linearly detrend version as this measure of marginal cost. We then estimate the response of this marginal cost to a monetary contractionary shock using both Smooth Local Projection[8] and regular Local Projection. We use the Wieland and Yang [2020] extended series of Romer and Romer [2004] monetary shocks series as instrument for movements in the fed funds rate. Figure 1 shows that the marginal cost responds positively to the monetary contractionary shock, pointing towards the dominance of the cost channel of monetary policy. Of course, this piece of evidence is very model-dependant and accordingly we will later explore more robust evidence by directly estimating the implied Phillips curves given by (3), without taking literally the restrictions on γ_y and γ_r implied by our specific micro-foundations.

1.3 Implications for Monetary Policy

We first need to emphasize that, as long as monetary policy is conducted in a way that maintains equilibrium determinacy, the Patman condition does not generally affect how the economy qualitatively responds to either demand or supply (markup) shocks. In particular, for a large class of monetary rules that includes standard Taylor rules, it is easy to show that demand shocks will always cause both activity and inflation to rise regardless of whether the Patman condition is met or not. Similarly, cost push shocks will lead to higher inflation and lower activity regardless of whether this condition is met. This observation is very important as it implies that the relevance – or irrelevance – of the Patman condition can not be evaluated by simply examining the qualitative properties of how the economy reacts to such shocks. To see this, the easiest is to consider demand shocks and markup shocks sequentially. For demand shocks (d shocks), if the response of monetary authorities is to increase real interest rates but do not over-compensate by causing a fall in activity (which is the case for a large set of policy rules including the form $i_t = E_t[\pi_{t+1}] + \phi_d d_t$ with $\phi_d < \frac{1}{\alpha_r}$), then the demand shock will lead to an increase in both inflation and output regardless of whether the T.E. Patman condition is met or not. For markup shocks (μ shocks), if the response of monetary

[8] See Barnichon and Brownlees [2019]. We have used the code they kindly provide.

Figure 1: Response of the model-consistent marginal cost to a one standard deviation contractionary monetary shock



Notes: The dark thick line is the response for the smooth local projection, the thick grey line for the local projection and the shaded area represents the 68% confidence band for the smooth local projection. The sample spans 1971Q2 to 2007Q4. Marginal cost measurement is derived from the model. We use the Wieland and Yang [2020] extended series of Romer and Romer [2004] monetary shocks series as instrument for movements in the fed funds rate.

authorities is to increase real interest rates but do not over-compensate by leading to a fall in inflation (which again is the case for a large set of rules), then the shock will lead to both an increase in inflation and a fall in activity regardless of whether the T.E. Patman condition is met or not.

We now turn to deriving implications of how different monetary stances affect the properties of the system given by equations (1), (2) and (3). In particular, we will want to emphasize how traditionally prescribed anti-inflationary responses to shocks can have qualitatively different effects on inflation depending on whether the Patman condition is met or not. Starting from the steady state, we consider a period 0 demand or markup shock of arbitrary persistence. The policy response is to increase the real interest rate to the level r in period 0, with persistence ρ_r, so that $r_t = \rho_r^t r$. Combining equations (1) and (2) (for arbitrary processes for μ_t and d_t), we can obtain the following equilibrium (π_t, r) locus::

$$\begin{aligned}\pi_t =& \kappa \frac{\rho_r^t}{1-\rho_r\beta}\left((\gamma_r - \alpha_r\gamma_y) - \frac{\rho_r}{1-\rho_r\alpha_y}\alpha_y\alpha_r\gamma_y\right) r \\ &+ \kappa\gamma_y \sum_{j=0}^{\infty}\beta^j\left(E_t\sum_{k=0}^{\infty}\alpha_y^k E_{t+j} d_{t+j+k}\right) + \sum_{j=0}^{\infty}\beta^j E_t \mu_{t+j}.\end{aligned} \tag{5}$$

Using equation (5), we can state Proposition 1.

Proposition 1. *If the T.E. Patman condition is not met, in response to either a positive supply shock or demand shock, engineering a rise in real interest rates will bring inflation closer to its target relative to keeping interest rates at their steady state value. If the T.E. Patman condition is met, a not too persistent rise in the interest rate will push inflation further away from its target relative to keeping real rates at their steady state value. By not too persistent, we mean* $\rho_r < \frac{(\gamma_r - \alpha_r\gamma_y)}{\alpha_y\gamma_r}$. *We will refer to this condition as the General Equilibrium (G.E.) Patman condition.*

Definition 2. General Equilibrium Patman Condition: $\rho_r < \frac{(\gamma_r - \alpha_r\gamma_y)}{\alpha_y\gamma_r}$

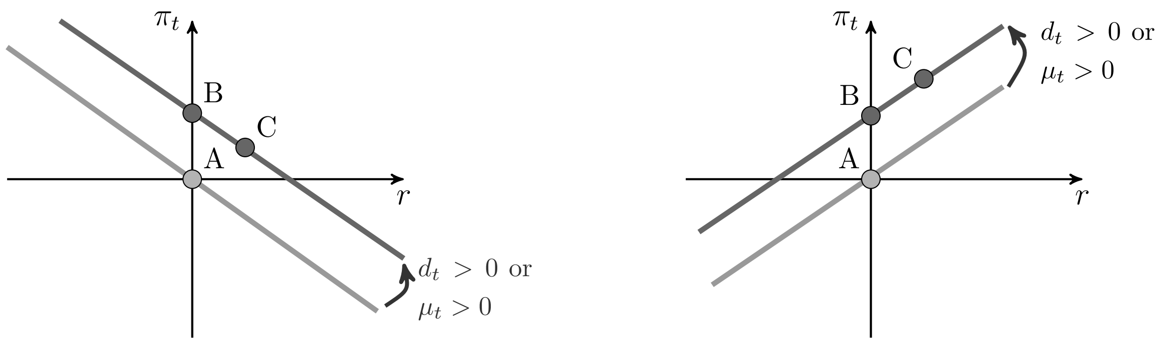
The proof of Proposition 1 is in A. Note that the G.E. Patman condition does not hold if the T.E. Patman condition fails. This proposition highlights that adding a cost channel does not alter how monetary policy can be used to help stabilize inflation if the Patman condition is not met. The first part of this proposition is illustrated on panel (a) of Figure 2, where in this figure we plot the (π_t, r) relation that we derived earlier. The important property of the resulting relationship between π_t and r is that it is negatively sloped, as it is given by $(\gamma_r - \alpha_r\gamma_y) - \frac{\rho_r}{1-\rho_r\alpha_y}\alpha_y\alpha_r\gamma_y$. Moreover, both demand and supply shocks shift this curve upwards. Therefore, in the absence of any move in interest rates, positive shocks to either demand or supply will increase inflation. Since the slope of the curve is negative, an increase in real interest rates will act to reduce inflation. The resulting increase in interest rates will help bring inflation closer to its target.

Panel (b) of Figure 2 corresponds to an economy in the Patman zone, for which the slope of the (π_t, r) locus $(\gamma_r - \alpha_r\gamma_y) - \frac{\rho_r}{1-\rho_r\alpha_y}\alpha_y\alpha_r\gamma_y$ is positive, which is true if the shock is not too persistent or equivalently if the G.E. Patman condition holds.[9] The (π_t, r) locus still

[9]Contrarily to the T.E. Patman condition that keeps expectations fixed, the G.E. Patman condition takes into account the endogeneity of expectations.

Figure 2: π_t as a Function of r in equilibrium

(a) T.E. Patman Condition does not hold (b) G.E. Patman Condition holds



Notes: equilibrium relationship between π_t and r as implied by Equation (5) when the T.E. Patman condition is not met (panel (a)) or is met with not too persistent increase in the real interest rate, meaning thet the G.E. Patman condition holds (panel (b)). Point A is the steady state, in which the economy was supposed to be before period t. A positive supply $\mu_t > 0$ or demand d_t shock shift the curve upwards. Point B corresponds to a shock $\mu_t > 0$. Considering monetary policy, point B represents keeping real interest rate at its steady state level, point C represents an interest rate increase in response to the supply shock.

shifts upwards with demand or markup shocks. In that case, increasing the real interest following a positive demand or markup shock will move inflation further away from target. The intuition for why a standard anti-inflationary prescription might destabilize inflation instead of helping stabilize it is rather evident. Recall that the Patman condition relates to the property that the direct effect of an increase in interest rates is larger than the indirect effect. Hence, in the Patman zone, increases in interest rates have the opposite effect than what is traditionally predicted. The traditional assumption is that the indirect effect always dominates the direct effect, with the later often assumed to be zero.[10] Note that when the economy is more likely to be in the Patman zone when the Phillips curve is flat – i.e., when γ_y is small. The cost channel γ_r needs not to be large in absolute value, what matters is the relative size of the two components of the marginal cost – i.e., $\frac{\gamma_r}{\gamma_y}$.

1.4 Missing Deflation and Low Inflation Trap

In this section we want to illustrate how a country can get trapped in a situation where simultaneously interest rates are at the Effective Lower Bound, inflation is below target and unemployment is below its steady state value. In particular, we want to emphasize how this situation can arise when monetary authorities depart from their traditional rules after a period of Effective Lower Bound constraint and low inflation – either to undo past inflation misses or simply to quickly bring inflation closer to its target.

To simplify exposition, we will assume that we are in a case where the Phillips curve is locally flat, so that γ_y is zero over a sufficiently wide interval around the steady state. This extreme assumption of a perfectly flat Phillips curve in a neighbourhood of the steady state

[10]In the canonical New Keynesian model there is not direct effect.

is not necessary for the point we want to make but it simplifies our presentation substantially. We also assume that the only shock present is an i.i.d. demand shock, again for clarity of exposition. Note that in this case, all expected terms will be zero.

Under the i.i.d. assumptions, inflation is simply given by:

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \gamma_r (i_t - E_t \pi_{t+1}),\\ &= \gamma_r i_t.\end{aligned}$$

We call this economy an extreme Patman economy. By contrast, the inflation equation in a standard New Keynesian economy will be:

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \gamma_y y_t,\\ &= \gamma_y y_t.\end{aligned}$$

In both economies, the Euler equation can be written as

$$\begin{aligned}y_t &= \alpha_y E_t y_{t+1} - \alpha_r (i_t - E_t \pi_{t+1}) + d_t,\\ &= -\alpha_r i_t + d_t.\end{aligned}$$

Finally, assume that the traditional monetary stance is to decrease real interest rates when demand falls, so that the desired policy rate would be to set:

$$i_t^d = E_t \pi_{t+1} + \psi_d d_t, \quad \psi_d > 0. \tag{6}$$

However, this policy is constrained by the Effective Lower Bound, which we denote by $\underline{i}$, so that the policy nominal rate is :

$$i_t = \max \left\{ i_t^d, \underline{i} \right\}. \tag{7}$$

Since variables are expressed in deviation from their steady state value, $\underline{i} < 0$.

Missing deflation Suppose the extreme Patman economy faces a temporary demand shock $-d < 0$ in period t and the monetary authorities follow the policy in (6) and (7). There is a threshold $\overline{d} = -\underline{i}/\psi_d$ such that the Effective Lower Bound constraint will bind if and only if $d > \overline{d}$.

We consider an econometrician that estimates a (mispecified) Phillips curve $\pi_t = \widehat{\gamma}_y y_t + \varepsilon_t$ using the demand shock d_t as an instrument.

First assume that the demand shock $d_t = -d < 0$ is not too severe – i.e., $d < \overline{d}$. The Effective Lower Bound will not be binding and monetary authorities will decrease the interest rate to the level $i_t = -\psi_d d$, so that $y_t = -(1-\alpha_r \psi_d) d$ and $\pi_t = -\gamma_r \psi_d d$. Using demand shocks as an instrument, the IV estimated slope of the Phillips curve will be $\widehat{\gamma}_y^N = \frac{\partial \pi_t}{\partial y_t} = \frac{\gamma_r \psi_d}{1-\alpha_r \psi_d}$, where N indicates that the Effective Lower Bound is *Not* binding. Note that this estimated slope is a function of the monetary policy stance ψ_d.

Assume now that the demand shock is negative enough for the Effective Lower Bound constraint to be binding – i.e., $d > \overline{d}$. Then monetary policy will be $i_t = \underline{i}$, so that $\pi_t = \gamma_r \underline{i} = -\gamma_r \psi_d \overline{d}$ and $y_t = \alpha_r \underline{i} - d = -(1 - \alpha_r \psi_d)\overline{d} - (d - \overline{d})$. In this case, the IV estimated slope of the Phillips curve is $\widehat{\gamma}_y = \frac{\partial \pi_t}{\partial y_t} = \frac{\gamma_r \psi_d}{1-\alpha_r \psi_d + (d-\overline{d})/\overline{d}} < \widehat{\gamma}_y^N$, as $d > \overline{d}$. As we can see, using again the demand shock as an instrument, the IV estimated slope of the Phillips curve is flattening out when the Effective Lower Bound binds. In contrast, in a standard New Keynesian model, the IV estimated slope of the Phillips curve will be constant and equal to γ_y.

In an extreme Patman economy, the period of mild deflation at the Effective Lower Bound could easily be mis-interpreted as an episode of missing deflation. In particular, if the monetary authority uses the past (linear) historical relationship between inflation and activity to predict how inflation should react in this episode, the fall in inflation when hitting the Effective Lower Bound would be smaller than predicted. This reflects the fact that, when the Effective Lower Bound is not constraining, inflation falls less in proportion to the demand shock than in normal times. This is the case because interest rates cannot fall as much. Therefore, when there is a cost channel to monetary policy, a period of perceived missing deflation at the Effective Lower Bound is readily explained.

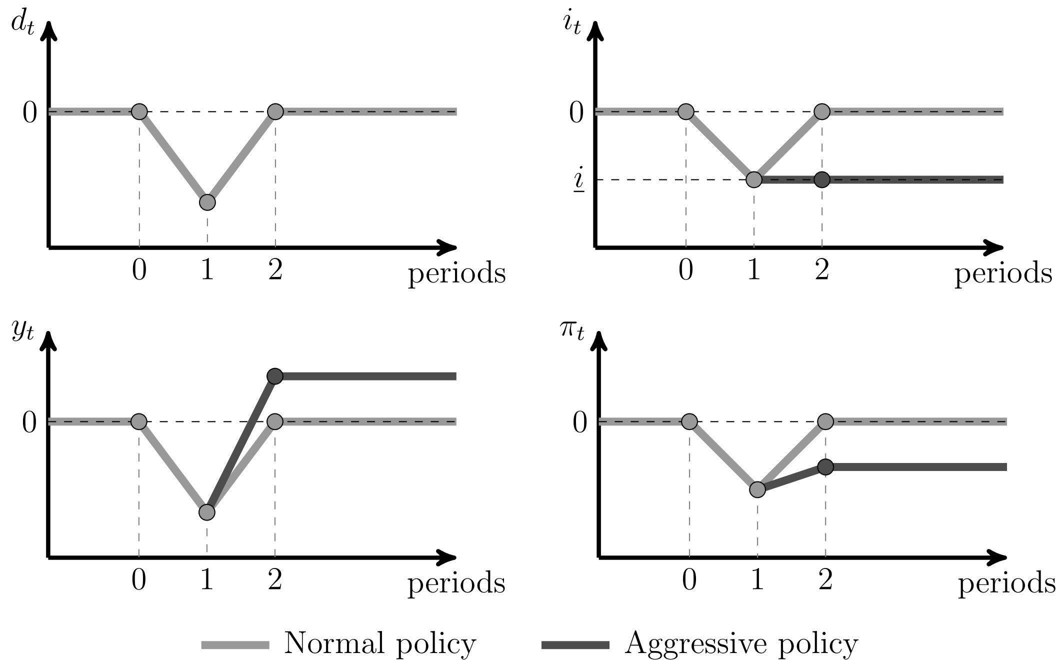
Low Inflation Trap : Let us now consider a slightly different policy stance which can lead to poorer inflation outcomes despite looking more aggressive by design. Such a policy will be very much in line with the one suggested by Ben Bernanke in a blog post on the Brookings website:

> "To be more concrete on how the temporary price-level target would be communicated, suppose that, at some moment when the economy is away from the ZLB, the Fed were to make an announcement something like the following:
> - The Federal Open Market Committee (FOMC) has determined that it will retain its symmetric inflation target of 2 percent. The FOMC will also continue to pursue its balanced approach to price stability and maximum employment. In particular, the speed at which the FOMC aims to return inflation to target will depend on the state of the labor market and the outlook for the economy.
> - The FOMC recognizes that, at times, the zero lower bound on the federal funds rate may prevent it from reaching its inflation and employment goals, even with the use of unconventional monetary tools. The Committee therefore agrees that, in future situations in which the funds rate is at or near zero, a necessary condition for raising the funds rate will be that average inflation *since the date at which the federal funds rate first hit zero be at least 2 percent*. Beyond this necessary condition, in deciding whether to raise the funds rate from zero, the Committee will consider the outlook for the labor market and whether the return of inflation to target appears sustainable."
> (Bernanke [2017], italics added by Bernanke)

We model such an idea in the following way. Consider the extreme Patman economy described in the previous paragraph. The desired policy stance is assumed to remain $i_t^d = E_t\pi_{t+1} + \psi_d d_t$ in normal times. However, normal times are here defined in a slightly stricter way than previously. They correspond to either *(i)* when the interest rate was not at the Effective Lower Bound last period or *(ii)* when the interest rate was at the Effective Lower Bound last period but $\pi_{t-1} \geq 0$. In abnormal times, when both $i_{t-1} = \underline{i}$ and $\pi_{t-1} < 0$, the rule is to set interest rates at the Effective Lower Bound, $i_t = \underline{i}$. Policy is then given by:

$$i_t = \begin{cases} \max\left\{\psi d_t, \underline{i}\right\} & \text{in normal times – i.e., when } [i_{t-1} > \underline{i}] \text{ or } [i_{t-1} = \underline{i} \text{ and } \pi_{t-1} \geq 0], \\ \underline{i} & \text{if } [i_{t-1} = \underline{i} \text{ and } \pi_{t-1} < 0]. \end{cases}$$

Figure 3: Inflation Trap with an Aggressive Monetary Policy



Notes: this figure plots responses of the nominal policy rate, the output gap and inflation to a negative demand shock that occurs in period 1 and puts the economy at the Effective Lower Bound. In each panel, the light line corresponds to a normal policy while the dark one represents the aggressive policy stance. See main text for the definition of those two policies. With the aggressive policy, equilibrium values of inflation and activity/unemployment after the demand shock has dissipated are, for $t \geq 2$, $\pi_t = \frac{\gamma_r}{1-\beta+\gamma_r}\underline{i} < 0$, $i_t - \pi_t = \frac{1-\beta}{1-\beta+\gamma_r}\underline{i} < 0$ *and* $y_t = \frac{-\alpha_r}{1-\alpha_y}\frac{1-\beta}{1-\beta+\gamma_r}\underline{i} > 0$.

This policy corresponds to keeping interest rates lower than standard policy when the economy has recently been at the Effective Lower Bound and inflation has been below target. From a standard perspective, this approach may seem aggressive as it is potentially correcting for low inflation episodes by keeping interest rates at the Effective Lower Bound even if the state of the economy would push the standard policy stance to increase interest rates. This type of policy can lead to situations where, in the absence of any new shocks, the policy rate gets stuck at the Effective Lower Bound even after the negative demand shock that initiated the Effective Lower Bound episode has dissipated.[11] Such a situation is plotted in Figure 3, where we display responses of the nominal policy rate, the output gap and inflation to a

[11] Here we exhibit an equilibrium in which agents expect the Effective Lower Bound to be binding forever after the initial shock under the aggressive policy, which happens in equilibrium. We do not claim that this is the unique equilibrium.

negative demand shock that occurs in period 1 and puts the economy at the Effective Lower Bound. When the above described aggressive policy is followed, inflation is stuck below target and unemployment is above its steady state value.[12] The economy could potentially remain stuck in such a low inflation trap until a sufficiently big supply shock pushes inflation up and leads to a re-normalization of policy.[13]

1.5 Monetary Shocks

Up to now, we have focused on the effects of the systematic part of monetary policy rules and we have not considered the effects of pure monetary shocks. To look at this issue, it is preferable to extend the model slightly to allow for some internal dynamics in order not to focus on knife edge cases. The easiest way to do this is to allow for external habit in consumption. In this case, the Euler equation for consumption takes the form[14]

$$y_t = \alpha_{y,f}\ E_t y_{t+1} + \alpha_{y,b}\ y_{t-1} - \alpha_r (i_t - E_t \pi_{t+1}) + d_t.$$

Now consider a monetary shock that aims to increase real rates for a while. For instance, this would be the case of an interest rate rule of the form $i_t = E_t \pi_{t+1} + \nu_t$, $\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_{\nu t}$, where $\varepsilon_{\nu t}$ is the monetary shock. In this case, it is clear that a tightening of monetary policy will lead to a persistent decline in economic activity as long as either ρ_ν or $\alpha_{y,b}$ are not equal to zero.

Inflation response to such a monetary shock will potentially cause the emergence of a price puzzle – i.e., inflation can rise on impact after a monetary contraction before declining below zero at later dates.[15] The occurrence of a price puzzle following a monetary shock is not surprising in environments in the Patman regime. However, the more interesting observation is that the length of the price puzzle will vary depending on both the persistence of the shock (ρ_ν), the size of the shock and the extent of internal dynamics – i.e., the size of $\alpha_{y,b}$. In order to get a better sense of these forces, it is helpful to consider the effects of a temporary change in interest rates of size r occurring at time 0. In this case, inflation for $t \geq 0$ is given by:

$$\pi_0 = \left(\gamma_r - \gamma_y \alpha_r \sum_{i=0}^{\infty} (\beta\lambda)^i \right) r,$$

$$\pi_t = -\gamma_y \alpha_r \lambda^t \left(\sum_{i=0}^{\infty} (\beta\lambda)^i \right) r,$$

[12] One of the reasons monetary authorities may be tempted by this policy is the fear that inflation becomes unanchored after a period of low inflation at the Effective Lower Bound. However, if the Patman condition is met, it is precisely following this policy that might trigger a de-anchoring of inflation expectations.

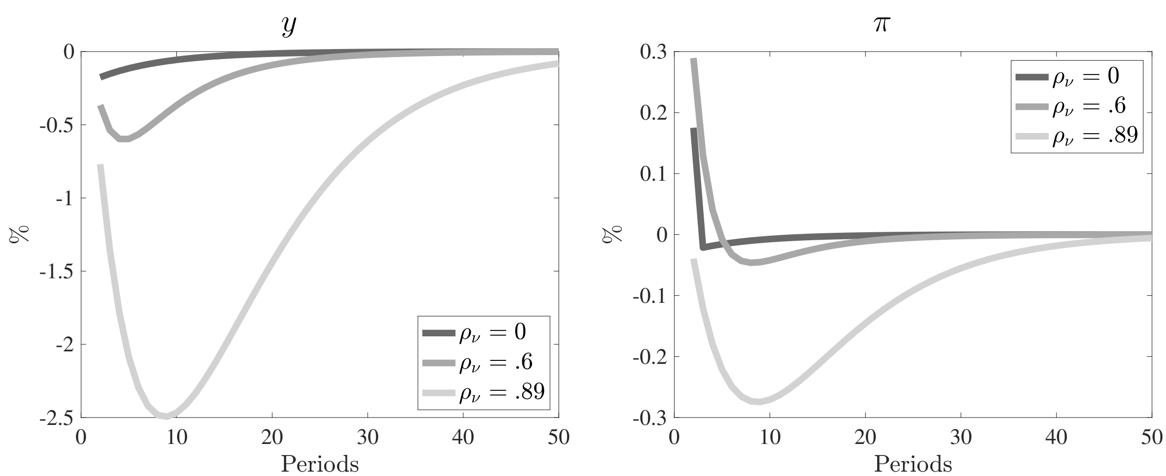
[13] Note that even if in this example we have a policy prescription somewhat similar to those associated with Neo-Fisherian view, the mechanism is very different. In particular, in the current framework, the inflation trap can arise even if inflation expectations remain well anchored. The main mechanism is not through expectations but through the cost channel.

[14] To avoid a unit root associated with real interest rate rules, we are again assuming that $1 - \alpha_{y,f} - \alpha_{y,b}$ is greater than zero but can be arbitrarily close to zero.

[15] We are aware that the price puzzle could be an artefact of poor controls for the Fed's information set. Our reading of the literature is that the jury is still out on wether the price puzzle is a fact or an artefact.

where λ is the stable root of the polynomial $\alpha_{y,f}X^2 - X + \alpha_{y,b}$. Here there would be a price puzzle in period 0 if $\gamma_r - \gamma_y \alpha_r \sum_{i=0}^{\infty} (\beta\lambda)^i$ is greater than zero. The condition $\left(\gamma_r - \gamma_y \alpha_r \sum_{i=0}^{\infty} (\beta\lambda)^i\right) > 0$ is the natural extension of the G.E. Patman condition for the case when there is external habit. Note that the basic T.E. Patman condition $\alpha_r \gamma_y < \gamma_r$ is a necessary condition for the price puzzle but is not sufficient. With a purely temporary increase in r, the price puzzle lasts one period in this case. After one period, inflation drops below steady state inflation and then converges back to its steady state value from below. In Figure 4, this response is displayed in dark grey. We also plot responses to a mildly persistent and very persistent shock. When persistence is mild, one observes several periods of "price puzzle" while there are none in the case of a more persistent shock. Note that output gap decreases in all scenarios.

Figure 4: Impulse Responses to a Monetary Shock when the G.E. Patman Condition Holds (for Various Persistence of the Shock, Linearized Model)



Notes: This shows the response of y and π to a 1% shock to the real interest rate. The model is linearized. Solutions are $y_t = \lambda_1 y_{t-1} - \frac{\alpha_r}{\alpha_{y,f}} \frac{\rho_\nu^t}{1-\rho_\nu \lambda_2^{-1}}$ and $\pi_t = \gamma_y \sum_{j=0}^{\infty} \beta^j y_{t+j} + \gamma_r \frac{\rho_\nu^t}{1-\rho_\nu \lambda_2^{-1}}$. The parameters values for these responses are $\beta = .99$, $\alpha_{y,f} = \alpha_{y,b} = .99/2$, $\alpha_r = .1$, $\gamma_y = .02$, $\gamma_r = .2$ and $\rho_\nu \in \{0, .6, .89\}$.

1.6 Non-Linear Model

It is beyond the scope of this paper to solve and estimate a non-linear version of our model. However, it is worth noting that the Patman configuration– if satisfied– should be thought as a local phenomena, applicable only near the steady state. The slope of the Phillips curve γ_y may be very close to zero (or equal to zero) when one is near the steady state of the system, and the Patman condition can be satisfied. However, when the economy deviates far from the steady state, it may be that the Phillips curve slope γ_y increases causing the parameterization to switch from Patman to a more regular case in which tight monetary policy decreases inflation. To illustrate this possibility, assume for simplicity that $\gamma_y = \widetilde{\gamma}_y y_t^2$ –as to represent that the effect of market tightness on wages may be more operative when far from the steady state– then the Phillips curve will take the form:

$$\pi_t = \beta E_t[\pi_{t+1}] + \widetilde{\gamma}_y y_t^3 + \gamma_r(i - E_t[\pi_{t+1}])).$$

Now suppose that monetary policy was of the form $i_t = E_t[\pi_{t+1}] + \phi_d d_t$, and for complete simplicity, assume that the demand shock d_t is an i.i.d. process and the only shock in the economy. In such a case, equilibrium inflation will be given by:

$$\pi_t = \gamma_\ell(1 - \alpha_r\phi_d)^3 d_t^3 + \gamma_r\phi_d d_t.$$

In such a model, a more activist monetary policy (higher ϕ_d) destabilizes inflation in the Patman zone and stabilizes it outside the Patman zone. Hence, if the demand shock distribution has a large variance, then activist policy may help stabilize inflation while if the shocks are not too large, it could destabilize inflation. Alternatively, in such a framework, one may want to choose a monetary policy that reacts very differently to small versus large shocks.

2 Estimating the Phillips Curve with Unrestricted Cost Channel

In this section, we explore properties of the New Keynesian Phillips curve when interest rates are allowed to directly affect real marginal costs. We do so by using the limited information-single equation approach initiated in the New Keynesian literature by Roberts [1995] and Galí and Gertler [1999].[16] While there is a substantial body of literature that allows for a monetary policy cost channel, most papers impose parameter restrictions which rule out by assumption the Patman configuration that is of interest to us. Therefore, our objectives in this section are twofold. First, we want to examine, within the confines of the New Keynesian Phillips Curve, whether interest rates have significant direct effects on inflation. Second, and most importantly, we want to look at whether the direct channel of monetary policy on inflation – i.e., the direct effect of interest rates – is large in comparison to the more standard indirect channel – i.e., working through market tightness.

2.1 Baseline Estimation

According to the first order approximation of the model derived in Section 1, the Phillips curve takes the form[17]:

$$\pi_t = \beta\pi_{t+1}^e + \gamma_y x_t + \gamma_r(i_t - \pi_{t+1}^e) + \mu_t, \tag{8}$$

where as before π_{t+1}^e is expected inflation, x_t is a measure of market tightness, i_t represents the nominal interest rate and μ_t is a markup shock.[18] Note that all the variables are demeaned, so that there is no constant in the equation.

[16]See the surveys of Nason and Smith [2008] and Mavroeidis, Plagborg-Møller, and Stock [2014].

[17]Notice that we normalize the coefficient attached to the marginal cost, $\kappa = 1$, as it is not separately identifiable from γ_r and γ_y. However this is not restrictive for our case as the value of κ is irrelevant when considering Patman condition, which is about the ratio $\frac{\gamma_y}{\gamma_r}$

[18]See Appendix H for an estimation of a "hybrid" version of the Phillips curve.

It is worth immediately noting that, from an estimation point of view, the distinction between whether one should allow real interest rates or nominal interest rates in this equation is irrelevant. Both lead essentially to the same regression up to a recombination of terms. We will return to this point later when discussing the interpretation of coefficients.

The biggest challenge in estimating the Phillips curve (8) relates to the endogeneity of the regressors. In our case, the endogeneity problem is compounded by the fact that we allow for interest rates to have a direct effect on inflation, knowing very well that the setting of interest rates is likely responding to inflation. For this reason, in all our estimations we will treat output gap, inflation expectations and interest rates as endogenous and follow Barnichon and Mesters [2020] in using identified monetary policy shocks as instruments. In particular, we will use six lags of the monetary policy shocks isolated in Romer and Romer [2004] and their squares as instruments.[19]

There are many data choices associated with estimating Equation (8). We will proceed in the following way. We use the U.S. Congressional Budget Office unemployment gap as our measure of market tightness. For our measure of the interest rate we use the Federal Funds Rate. For expected inflation we use the "Expected Change in Prices During the Next Year" of the Michigan Survey of Consumer expectations[20,21] or we assume full information rational expectation (FIRE) [22].

First pass estimations. For inflation, we use as headline CPI inflation as a first pass, and control for oil price in the estimation. The advantage of this choice is that we can use a long sample that starts in 1969. We first estimate the Phillips curve without including the cost channel of inflation (columns (1), (3) and (5) of Table (1). The slope of the Phillips curve γ_y is positive, significant at 1% with the survey measure of expectations, not at 10% otherwise. The clear and consistent results we obtain is that, once we also include the interest rate, the cost channel γ_r is always positive and significant, while the slope γ_y becomes smaller, always insignificant, with a point estimate that is not always positive, which points towards a Patman regime.

Table 2 presents a set of of robustness check relative to the data choices made in Table 1. A larger set of robustness checks is available in Beaudry, Hou, and Portier [2020]. In the first two columns of Table 2 we use the CBO output gap instead of the unemployment gap as our measure of economic slack. As can be seen, results are very similar to those in Table 1 with the interest rate effect continuing to enter our estimated Philips curve significantly.

[19] The original Romer and Romer [2004] shocks series ends in 1996. We instead use the shocks series extended to 2007 by Wieland and Yang [2020].

[20] See Coibion, Gorodnichenko, and Kamdar [2018] for a recent overview of the literature that uses survey data in the estimation of Phillips curves.

[21] In the Michigan Survey of Consumers, every month a representative sample of consumers are asked the following question: "*By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*" The answer to this question is then the one-year-ahead inflation expectation $E_t\pi_{t+4,t}$. To keep consistency with the quarter-to-quarter inflation we use in the estimation, we rescaled the one-year-ahead expected inflation assuming survey respondents believe that quarter-to-quarter inflation follows an AR(1) process with persistence $\tilde{\rho}$, that needs not to be equal to the actual persistence of inflation. See F for details.

[22] Note that the empirical literature on inflation expectations document prominent evidence on deviations from FIRE (see for example Coibion and Gorodnichenko [2015]).

Table 1: First Pass Estimation of the Phillips Curve using Headline CPI Inflation

π^e	MSC		FIRE	
	(1)	(2)	(3)	(4)
β	1.12	1.18	0.81	0.98
	(0.079)	(0.074)	(0.098)	(0.106)
γ_y	0.12	0.06	0.08	-0.07
	(0.047)	(0.053)	(0.071)	(0.076)
γ_r		0.14		0.21
		(0.041)		(0.062)
Observations	150	150	150	150
J Test	7.607	8.515	5.538	5.919
(jp)	(0.815)	(0.667)	(0.938)	(0.879)
Weak ID Test	3.387	3.091	1.804	1.643

Notes: All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. The measure of inflation is Headline CPI Inflation, the measure of market tightness is the U.S. Congressional Budget Office unemployment gap. We use the Michigan Survey of Consumers to measure inflation expectations is the MSC columns, and assume Full Information Rational Expectations in the FIRE ones. Real oil price is added as a control in all the equations and all regressors are instrumented using six lags of Romer and Romer [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. For γ_y and γ_r, estimates highlighted in grey are significant at 1% and not significant at 10% if not highlighted. Sample is 1969Q1-2007Q4.

In the next two columns we replace Headline CPI inflation by Core CPI as our measure of inflation and no longer control of oil prices in estimation. Results are again robust to this modification and provide further support in the direction of the Patman condition.

Table 2: First Pass Estimation of the Phillips Curve, Robustness

π	HL CPI		Core CPI	
Gap	ygap		*minus* ugap	
π^2	MSC	FIRE	MSC	FIRE
	(1)	(2)	(3)	(4)
β	1.10	0.74	0.97	0.74
	(0.072)	(0.096)	(0.057)	(0.053)
γ_y	0.06	0.09	-0.05	0.00
	(0.025)	(0.039)	(0.053)	(0.059)
γ_r	0.17	0.21	0.19	0.47
	(0.038)	(0.058)	(0.066)	(0.065)
Observations	150	150	150	150
J Test	8.729	7.072	9.372	10.302
(jp)	(0.647)	(0.793)	(0.588)	(0.503)
Weak ID Test	5.015	2.629	2.865	2.734

Notes: All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. All regressors are instrumented using six lags of Romer and Romer [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. For γ_y and γ_r, ■ denotes significance level at 5%, □ denotes significance level at 1%. Sample is 1969Q1-2007Q4.

Preferred estimations using Core R-CPI inflation. Our preferred estimations use the BLS "Consumer Price Index retroactive series using current methods for all items less food and energy" (R-CPI-U-RS, or Core R-CPI for short). We use this series instead of the CPI because before 1983, the shelter component of the CPI was computed using, among other small components, an index of house prices, and an index of mortgage rates. Mortgage rates directly comove with the effective federal funds rate, and indirectly, through discount rates, house prices do too. This would mechanically make CPI inflation reacting to the federal fund rate. Since 1983, the BLS adjusted its methodology, and changed the computation of the shelter component of the CPI in favour of a rental equivalence index, including an owner occupied rental equivalence index. The BLS does not retroactively adjust the methodology in its price indexes. The advantage of the Core R-CPI is that such a retroactive adjustment is done. As this series is not seasonally adjusted, we use a year-to-year measure of inflation.[23]

Finally, it is well known that the slope of the Phillips curve (γ_y) is difficult to estimate using aggregate data (Mavroeidis, Plagborg-Møller, and Stock [2014]). The most recent and credible estimates exploit cross regional variations, as in Hazell, Herreño, Nakamura, and

[23] See Appendix G for details.

Steinsson [2020]. Therefore, to bypass controversies about γ_y, we repeat our estimation of the Phillips curve (8) imposing the slope parameter $\gamma_y = 0.0138$, as estimated by Hazell, Herreño, Nakamura, and Steinsson [2020].[24].

Our preferred set of results is presented in Table 3. Our baseline estimation is column (1), where we use expectations surveys and estimate both γ_y and γ_r: the slope of the Phillips curve γ_y is small and negative, while the real interest rate coefficient γ_r is positive and significant.[25] When we set γ_y to the estimated value of Hazell, Herreño, Nakamura, and Steinsson [2020] (column (2)), the cost channel parameter γ_r is still highly significant and positive, and larger than when γ_y is estimated. When we repeat the estimation assuming FIRE, we obtain similar results, although the size of the cost channel coefficient is smaller, but still highly significant. This set of results indicate that the US economy may likely be operating in the Patman zone.

2.2 Iterating Forward the Phillips Curve

The previous results are of interest because we use only identified monetary policy shocks as instrumental variables and these monetary shocks are strong instruments. However, there are also draw-backs in such identification strategy. The standard formulation of the Phillips curve imposes strong restrictions on the timing of inflation variations. The identification through monetary policy shocks works like decomposing the responses of inflation, expectation and real interest rate to these shocks. Empirically these variables may not respond to the monetary shocks simultaneously, and in the Phillips curve relation, current inflation may also respond to economic slackness and real interest rate with lags. Looking at it this way, the timing restriction may make these estimates problematic. We address this by following Hazell, Herreño, Nakamura, and Steinsson [2020] in iterating forward the Phillips curve. As explained in the previous pararagraph, we choose not to estimate the slope parameter γ_y and instead set it to the estimated value of Hazell, Herreño, Nakamura, and Steinsson [2020] as to focus on the role of the cost channel.

First we derive the long-run Phillips curve from Equation (8):

$$\pi_t = \sum_{j=0}^{\infty} \beta^j (\gamma_y E_t x_{t+j} + \gamma_r E_t r_{t+j} + E_t \mu_{t+j}) + \underbrace{\lim_{j \to \infty} \beta^j E_t \pi_{t+j}}_{=0} \tag{9}$$

Assume that x_t and r_t have a long-run and a transitory component so that $x_t = \widetilde{x}_t + x_\infty$ and

[24] In Hazell, Herreño, Nakamura, and Steinsson [2020], the authors provide an implied aggregate slope of the Phillips curve, with R-CPI as the measure of inflation and negative unemployment gap as the measure of market slackness. From footnote 22 in Hazell, Herreño, Nakamura, and Steinsson [2020], this aggregate slope of the Phillips curve is $= 0.58 \times 0.0062 + 0.42 \times 0.0243 = 0.0138$. Furthermore, although the authors didn't estimate a Phillips curve with real interest rate, they did control for time fixed effect in estimating the slope. If real interest rate is common across states, the impact of direct cost channel is taken care of by the time fixed effect. For these reasons, it is appropriate to use their estimates in our analysis.

[25] Estimates for γ_r are smaller than first-pass estimates because mortgage rates are directly used to compute inflation is the later.

Table 3: Baseline Estimation of the Phillips Curve using Core R-CPI

π^e	MSC		FIRE	
	(1)	(2)	(3)	(4)
β	0.91	0.95	0.98	1.01
	(0.020)	(0.019)	(0.007)	(0.005)
γ_y	-0.08	$0.0138^\dagger$	-0.03	$0.0138^\dagger$
	(0.018)	(–)	(0.007)	(–)
γ_r	0.12	0.20	0.06	0.10
	(0.031)	(0.014)	(0.010)	(0.003)
Observations	119	119	119	119
J Test	8.664	10.463	7.942	8.720
(jp)	(0.653)	(0.575)	(0.718)	(0.727)
Weak ID Test	10.648	119.574	13.246	115.405

Notes: All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. The measure of inflation is BLS "Consumer Price Index retroactive series using current methods for all items less food and energy", the measure of market tightness is the U.S. Congressional Budget Office unemployment gap. We use the Michigan Survey of Consumers to measure inflation expectations is the MSC columns, and assume Full Information Rational Expectations in the FIRE ones. In columns (1) and (3), we estimate the slope parameter γ_y, while it is fixed to the value estimated by Hazell, Herreño, Nakamura, and Steinsson [2020] in columns (2) and (4). All regressors are instrumented using six lags of Romer and Romer [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. † denotes a parameter value that is imposed and not estimated. For γ_y and γ_r, estimates highlighted in grey are significant at 1% and not significant at 10% if not highlighted. Sample is 1978Q2-2007Q4.

$r_t = \widetilde{r}_t + r_\infty$. We can then rewrite Equation (9) as:

$$\begin{aligned}\pi_t &= \gamma_y \sum_{j=0}^{\infty} \beta^j E_t \widetilde{x}_{t+j} + \gamma_r \sum_{j=0}^{\infty} \beta^j E_t \widetilde{r}_{t+j} + \frac{1}{1-\beta}(\gamma_y E_t x_\infty + \gamma_r E_t r_\infty) + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j} \\ &= \gamma_y \sum_{j=0}^{\infty} \beta^j E_t \widetilde{x}_{t+j} + \gamma_r \sum_{j=0}^{\infty} \beta^j E_t \widetilde{r}_{t+j} + E_t \pi_\infty + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j} \end{aligned} \tag{10}$$

The last equation follows from Equation (8), as when $t \to \infty$ we have $E_t\pi_\infty = \frac{1}{1-\beta}(\gamma_y E_t x_\infty + \gamma_r E_t r_\infty)$. We set the Phillips curve slope γ_y to the estimated value in Hazell, Herreño, Nakamura, and Steinsson [2020] and use year-to-year inflation rate. We truncate the infinite time horizon at $T = 40$, which is equivalent to ten years and use the ten-year ahead CPI forecast from Cleveland Fed as a measure of $E_t\pi_\infty$. We then use ten-year moving average to compute the long run component of real interest rate r_∞ and get $\widetilde{r}_t = r_t - r_\infty$. Then following again Hazell, Herreño, Nakamura, and Steinsson [2020], we use negative unemployment gap as $\widetilde{x}_t$. We can then estimate Equation (10) by replacing $\sum_{j=0}^{\infty} \beta^j E_t \widetilde{x}_{t+j}$ and $\sum_{j=0}^{\infty} \beta^j E_t \widetilde{r}_{t+j}$ with $\sum_{j=0}^{T} \beta^j \widetilde{x}_{t+j}$ and $\sum_{j=0}^{T} \beta^j \widetilde{r}_{t+j}$ and instrument with monetary shocks prior to time t. The sample we use here is `1982Q1-2007Q4` due to the availability of the ten-year ahead CPI forecast. Results are presented in Table 4. The key take away of Table 4 is that the estimate

Table 4: Estimation of Iterated Phillips Curve

γ_y	0.0138†
	(–)
γ_r	0.11
	(0.020)
Observations	104
J Test	3.677
(jp)	(0.994)
Weak ID Test	51.317

Notes: All the results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. Both expected inflation and real rates are instrumented withRomer and Romer [2004] shocks (as extended by Wieland and Yang [2020]). † denotes a parameter value that is imposed and not estimated. Estimates highlighted in grey are significant at 1%. Sample runs from `1982Q1` to `2007Q4`.

of γ_r is again positive, highly significant and of the same magnitude than in Table 3.

2.3 Nominal versus Real Interest Rates?

So far, we have presented theory and data that emphasize the impact of the real interest rate on the marginal cost and thereby on inflation. However, in most of the literature on the cost channel of monetary policy, it is the nominal interest rate that is highlighted to affect inflation. As noted previously, in terms of Phillips curve estimation, this distinction does not matter for the estimation of γ_r as both approaches lead to the same estimating equation. Indeed, the equation we have estimated is

$$\pi_t = \beta\pi^e_{t+1} + \gamma_y x_t + \gamma_r(i_t - \pi^e_{t+1}) + \mu_t,$$

and it can be rewritten as:

$$\pi_t = (\beta - \gamma_{r)}\pi^e_{t+1} + \gamma_y x_t + \gamma_r i_t + \mu_t$$

However, by focusing on estimated coefficients, especially the coefficient on expected inflation, one can get a sense of whether a real interest rate or a nominal interest rate interpretation is preferable. One of the implications of the New Keynesian Philips curve literature is that the coefficient of expected inflation should be close to agents' discount factor. Given the fact that we use quarterly data, this would suggest a coefficient of expected inflation close to .99. Taking our baseline estimation – looking at column (1) of Table 3 – we most obtain a coefficient β on expected inflation that is .91, which is already smaller than the hypothetical .99. However, if we were to adopt a nominal rate specification, then we would need to subtract the coefficient we found for real rates γ_r (which is around .12) from the coefficient β we estimated for expected inflation. This would imply a β around .8 instead of around .91. This would be quite far from theoretical predictions, which points towards the real rate specification.

We can also directly compare the real and nominal rate specification if we set $\beta = .99$ and $\gamma_y = 0.0138$ and estimate the two following equations:

$$\pi_t = .99\pi^e_{t+1} + 0.0138x_t + \gamma_r(i_t - \pi^e_{t+1}) + \mu_t, \quad (11)$$
$$\pi_t = .99\pi^e_{t+1} + 0.0138x_t + \gamma_r i_t + \mu_t. \quad (12)$$

Results of these estimations are displayed in Table 5. The cost channel parameter γ_r is significant in both specifications, but the R-squared is .25 for the real interest specification and only .02 for the nominal interest one; the real interest rate specification is unambiguously preferred by the data.

Table 5: Real versus Nominal Interest Rate

	With real interest rate Equation (11)	With nominal interest rate Equation (12)
β	$.99^{\dagger}$	$.99^{\dagger}$
	(–)	(–)
γ_y	$0.0138^{\dagger}$	$0.0138^{\dagger}$
	(–)	(–)
γ_r	0.20	0.09
	(0.022)	(0.013)
R^2	0.250	0.022

Notes: All the results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported between parentheses. The constant term is omitted from the table. Inflation is measured by the year-to-year core R-CPI and expectations are from the Michigan Survey of Consumers. The real and nominal rates are instrumented with the Romer and Romer [2004] shock series. † denotes a parameter value that is imposed and not estimated. Estimates highlighted in grey are significant at 1%. Sample runs from 1978Q2 to 2007Q4.

2.4 Some Further International Evidence

In a recent work, Coibion, Gorodnichenko, and Ulate [2019] estimate a New-Keynesian Phillips curve by pooling across a range of countries who have consumer or firm surveys available. They assemble time series of inflation expectations for 18 countries/regions (Australia, Canada, Chile, Czechia, Denmark, Finland, France, Germany, Israel, Italy, Japan, New Zealand, South Korea, Sweden, Turkey, United Kingdom, United States, as well as the entire eurozone) over different periods.[26] They found "*a robust and negative relationship between the inflation gap (the deviation of inflation from expected inflation) and the unemployment gap (the deviation of unemployment from the natural rate)*". Although the panel regression does not use instrumental variable, its merit is to allow for estimation over more than 1,000 country-quarter observations. The estimated equation (omitting the constant) is

$$\pi_{i,t} - \pi^e_{i,t+1} = \gamma_y y_{i,t} + c_i + \varepsilon_{i,t}, \tag{13}$$

where i is a country index, $y_{i,t}$ is minus the unemployment gap and c_i are country fixed effects. We use the same data plus a measure of the real interest rate to estimate a Phillips curve augmented with a cost channel. [27] The estimated equation is in this case

$$\pi_{i,t} - \pi^e_{i,t+1} = \gamma_y y_{i,t} + \gamma_r \left(i_{i,t} - \pi^e_{i,t+1}\right) + c_i + \varepsilon_{i,t}. \tag{14}$$

Estimation results are presented in Table 6. Two results emerge. First, the real interest

Table 6: The Expectations-Augmented Phillips Curve across Countries

	Equation (13)	Equation (14)
γ_y	0.32	.019
	(0.10)	(0.17)
γ_r	–	0.20
	(–)	(0.07)
Adj. R^2	0.63	0.66
Observations	1062	1062

Notes: See Coibion, Gorodnichenko, and Ulate [2019] for data description and estimation method. ■ denotes significance level at 5%, ■ denotes significance level at 1%. Coefficients that are not highlighted are not significant at 10%. Standard errors are between parentheses and are clustered at the country and quarter level.

enters positively and significantly (p-value is .9%), and the coefficient γ_r is pretty close to the ones we obtain in our US estimates (.2). Second, the slope of the Phillips curve γ_y is reduced by almost half (.19) and it looses significance (p-value is 17%). We find these results as an extra piece of evidence of the flatness of the Phillips curve and on the significance of the cost channel, and this points again towards the Patman zone.

[26]See Coibion, Gorodnichenko, and Ulate [2019] for a precise description of data and estimation method. We thank them for providing data and codes.

[27]See Appendix E for the choice of the nominal interest rate.

3 Structural Estimation

The goal of this section is to estimate our simple extended three-equation New Keynesian model, where we do not *a priori* take any stance on whether parameters satisfy the Patman condition. Our objective is to see whether the Patman parameterization may offer a better fit to the data than more standard parameterizations implicit in most New Keynesian models.

3.1 The Estimated Equations

The initial model we want to estimate includes the following two equations

$$\begin{aligned} y_t &= \alpha_y E_t[y_{t+1}] - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t, && \text{(EE)} \\ \pi_t &= \beta E_t[\pi_{t+1}] + \kappa\big(\gamma_y y_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, && \text{(PC)} \end{aligned}$$

where d_t and μ_t are assumed to be independent AR(1) processes. Here we are expressing market tightness by y, which should be interpreted as labor gap. Since in our simple framework the labor gap and the output gap are interchangeable, we chose to express market tightness by y to remind the reader of this property.

We choose to close the model with the following class of policy rules:

$$i_t = E_t[\pi_{t+1}] + \phi_d d_t + \phi_\mu \mu_t + \nu_t. \qquad \text{(Policy)}$$

This class of policy rules is attractive as it minimizes difficulties associated with indeterminacy while simultaneously being very flexible as it allows monetary policy to react to the state space of the system. With such a real rate rule, the equilibrium is determinate as long as $|\alpha_y| < 1$. In the baseline estimation, we assume quasi-no Euler discounting by setting $\alpha_y = .99$. Note that in this policy rule, ν_t will represent monetary shocks that we also assume to be AR(1). In D, we prove that for any monetary rule that reacts to current endogenous variables and that guarantees determinacy of equilibrium – which includes the typical Taylor rule estimated in the literature – equilibrium allocations can be replicated with our class of policy rules. Estimating a model with our policy is therefore not restrictive, and nests a Taylor rule specification.

The model is a simple linear system of three equations in three unknowns: y, π and i. As such, this system has low dimension, is entirely forward looking and is unlikely to fully capture the rich dynamics of the economy. An attractive feature of using such a simple model is that all the mechanisms at play can be understood easily. The drawback is that it may be an over-simplification. We believe that it is a useful starting point as it allows us to ask whether the simple narrative of a striped down New Keynesian model offers a better interpretation of the data than what could be offered by a Patman parametrization– a parameterization that is generally not considered in the literature. We will later introduce internal dynamics.

3.2 Estimation, Identification and Sample Period

We estimate the above model following a classical maximum likelihood method. As commonly done in the empirical macroeconomic literature, we calibrate some parameters. First, one cannot separately identify κ, γ_y and γ_r. Instead we can only get estimates of $\kappa\gamma_y$ and $\kappa\gamma_r$. Without loss of generality, we therefore normalize $\kappa = 1$. We set β to .99, which is in line with large parts of the literature. Our results are not sensitive to changing β around this level. We set α_y to .99, so that although there is almost no discounting in the Euler equation, the model is always determinate.

Data and sample are the same than in the baseline estimation of Section 2. All estimations are performed using DYNARE.[28]

3.3 Results

Table 7 presents the baseline estimation of our forward-looking sticky prices model.

Table 7: Estimated Parameters, Simple Model

α_r	0.01	γ_y	0.013	γ_r	0.034			
	(0.01)		(0.030)		(0.016)			
ϕ_d	0.46	ϕ_μ	-0.58	σ_d	0.02	σ_μ	0.51	
	(0.12)		(0.10)		(0.01)		(0.08)	
σ_ν	0.27	ρ_d	0.94	ρ_μ	0.40	ρ_ν	0.99	
	(0.14)		(0.03)		(0.09)		(0.01)	
T.E. Patman condition							0.034	(0.016)
G.E. Patman condition							0.089	(0.030)

Notes: this table shows the estimated coefficients of equations (EE), (PC) and (Policy) with unemployment gap, Core CPI Research Series. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. Standard errors are between parenthesis. Sample runs from 1978Q2 to 2007Q4. T.E. Patman condition corresponds to $\gamma_r - \alpha_r\gamma_y$, G.E. Patman condition is the impact response of inflation π to a one standard deviation monetary policy shock.

In this estimation, parameters ρ_d, ρ_μ and ρ_ν are restricted to be in the unit interval. The first thing to note from the table is that the signs of the estimates are the expected ones. Monetary policy is observed to increase interest rates in response to demand shocks ($\phi_d > 0$) and to decrease it in response to cost-push shocks ($\phi_\mu < 0$). The estimated value of α_r is very low. There is recent evidence from micro data obtained by Best, Cloyne, Ilzetzki, and Kleven [2020] pointing at a low intertemporal elasticity of substitution, although for consumption only. In I.1, we re-estimate the model constraining α_r to take the values estimated by Smets and Wouters [2007] and show that we obtain qualitatively the same results. Finally, the Phillips curve slope (γ_y) is not significantly different from zero and smaller than the real interest rate channel (γ_r), which is positive and significant. Henceforth, the T.E. Patman condition is clearly satisfied. As this is only a necessary condition in this model with persistent shocks, we also compute the G.E. Patman condition, which is given by the impact response of inflation to a one standard deviation monetary policy shock. As it can be checked in Table 7, that response is positive and significant at a 95% level. This

[28]See Adjemian et al. [2020]

simple model estimation confirms what we have found in the previous section: the Phillips curves appears very flat, but has a positive and significant cost channel.

Since this model of this section is purely forward looking, we now extend it to allow for some standard internal propagation mechanisms.

3.4 Extending the Model

We now consider an extended version of our baseline model where we allow for internal propagation mechanisms in the three equations. To that effect, we follow the literature and introduce habit persistence, hybrid Phillips curve and persistence in the policy rule. The derivation of the Euler equation and Phillips curve are presented in C.

The model now takes the form:

$$\begin{aligned} y_t &= \alpha_y\big(\alpha_{y,f}E_t[y_{t+1}] + (1-\alpha_{y,f})y_{t-1}\big) - \alpha_r(i_t - E_t[\pi_{t+1}]) + d_t, && \text{(EE')} \\ \pi_t &= \beta\big((1-\beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}\big) + \kappa\big(\gamma_y y_t + \gamma_{y,b}y_{t-1} + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, && \text{(PC')} \\ i_t &= E_t[\pi_{t+1}] + \phi_{r,b}\big(i_{t-1} - E_{t-1}[\pi_t]\big) + \phi_{\pi,b}\pi_{t-1} + \phi_{y,b}y_{t-1} + \phi_d d_t + \phi_\mu \mu_t + \nu_t. && \text{(Policy')} \end{aligned}$$

With habit persistence, past output also enters in the marginal cost. In order to facilitate comparison with the Phillips curve estimations of Section 2, we present estimates of the Phillips curve where we do not include lagged market tightness in the marginal cost – i.e., we use the Phillips Curve equation of the form:

$$\pi_t = \beta\big((1-\beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}\big) + \kappa\big(\gamma_y y_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, \qquad \text{(PC”)}$$

In I.3, we show that results are unaffected when we estimate the model with (PC’) rather than (PC”). The policy rule includes past real interest rate on top of all the states of the economy – i.e., $\{d_t, \mu_t, \nu_t, \pi_{t-1}, y_{t-1}\}$.

As we have five more parameters than in the simple model, a classical maximum likelihood method would become a nonlinear optimization problem that is quite unstable. We therefore perform a Bayesian estimation, as in this case the use of prior distributions over the structural parameters makes this optimization more stable. In I.2, we present the choice of priors and show detailed results such as parameters priors and posterior distributions. Table 8 presents the parameters estimates.

Parameters are well identified and have the expected sign. Interestingly, at the median of the posterior distribution, the slope of the Phillips curve γ_y is -.03 and not significantly different from zero while the cost channel γ_r is significant and equal to .07. Once again, the parameters configuration is such that the T.E. Patman condition is met.[29] Furthermore, the G.E. Patman condition is also satisfied, as the impact response of inflation to a one standard deviation monetary shock is postive and different from zero at a 95% level.

[29] In a previous version of this work (Beaudry, Hou, and Portier [2020]), we show that the results we found here are robust to choices of samples and measures of inflation.

Table 8: Estimated Parameters, Extended Model

α_r	0.02	γ_y	-0.03	γ_r	0.07	ϕ_d	0.51
	[0.01, 0.03]		[-0.11, 0.04]		[0.03, 0.11]		[0.30, 0.73]
ϕ_μ	-0.71	σ_d	0.04	σ_μ	0.38	σ_ν	0.28
	[-0.85, -0.58]		[0.03, 0.06]		[0.29, 0.48]		[0.17, 0.43]
ρ_d	0.85	ρ_μ	0.62	ρ_ν	0.94	β_b	0.05
	[0.79, 0.90]		[0.52, 0.72]		[0.91, 0.96]		[0.01, 0.11]
$\phi_{\pi,b}$	0.01	$\phi_{r,b}$	0.14	$\alpha_{y,f}$	0.75	$\phi_{y,b}$	0.09
	[-0.13, 0.14]		[-0.01, 0.28]		[0.66, 0.88]		[-0.23, 0.42]
T.E. Patman condition			0.07	[0.03, 0.11]			
G.E. Patman condition			0.10	[0.07, 0.13]			

Notes: This table shows the posterior median estimates of the coefficients in equations (EE'), (PC") and (Policy') using unemployment gap, Core CPI Research Series and using the sample 1978Q2-2007Q4. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chains. The numbers between brackets represent the 95% confidence band using the posterior distribution. Sample runs from 1978Q2 to 2007Q4. T.E. Patman condition corresponds to $\gamma_r - \alpha_r \gamma_y$, G.E. Patman condition is the impact response of inflation π to a one standard deviation monetary policy shock.

4 Conclusion

During the last two decades prior to the Covid-19 pandemic, the behavior of inflation has been puzzling in several countries. First, during 2008-09 recession, inflation fell by less than anticipated given the depth of the recession. This became known as the missing deflation puzzle. After that, the puzzle reversed with inflation generally remaining below target in many countries despite the experience of historically low rates of unemployment. This in turn became known as the missing inflation puzzle. Both these puzzles could reflect a relatively flat Phillips curve. This paper builds on this observation and goes a step further by exploring the monetary policy implications of a quite flat Phillips curve when a cost channel of monetary policy may also be present. We show how standard prescriptions for monetary policy may need to be modified in such an environment. In particular, to keep inflation close to its target in face of positive demand and markup shocks, we argue that a central bank may want to keep real interest rates unchanged or to decrease them. One of the interesting features of this framework is that it offers a simple explanation to why and when a country may find itself trapped for a considerable amount of time at the Effective Lower Bound with inflation below target and employment above its steady state value. A large part of the paper has been devoted to show that the condition under which these features arise are supported in US data.

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Appendix to
"Monetary Policy when the Phillips Curve is Quite Flat"

Paul Beaudry, Chenyu Hou, Franck Portier

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A Proof of Proposition 1

We assume that $r_t = \rho_r^t r$. Solving forward the Euler equation (2) with the marginal cost given by (3), we obtain

$$y_t = -\frac{\alpha_r}{1-\rho_r\alpha_y}\rho_r^t r + \sum_{j=0}^{\infty} \alpha_y^j E_t d_{t+j},$$

which implies that

$$y_{t+j} = -\frac{\alpha_r}{1-\rho_r\alpha_y}\rho_r^{t+j} r + \sum_{k=0}^{\infty} \alpha_y^k E_t d_{t+j+k}.$$

Take now the Phillips curve and solve forward to obtain

$$\pi_t = \kappa\gamma_y \underbrace{\sum_{j=0}^{\infty} \beta^j E_t y_{t+j}}_{\mathcal{A}} + \kappa\frac{\gamma_r}{1-\rho_r\beta}\rho_r^t r + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j},$$

with

$$\mathcal{A} = -\frac{\alpha_r}{(1-\rho_r\alpha_y)(1-\rho_r\beta)}\rho_r^t r + \sum_{j=0}^{\infty} \beta^j \left(\sum_{k=0}^{\infty} \alpha_y^k E_t d_{t+j+k}\right).$$

We therefore obtain the (π_t, r) equilibrium locus

$$\begin{aligned}\pi_t =& \kappa\frac{\rho_r^t}{1-\rho_r\beta}\left((\gamma_r - \alpha_r\gamma_y) - \frac{\rho_r}{1-\rho_r\alpha_y}\alpha_y\alpha_r\gamma_y\right) r \\ &+ \kappa\gamma_y \sum_{j=0}^{\infty} \beta^j \left(E_t \sum_{k=0}^{\infty} \alpha_y^k E_{t+j} d_{t+j+k}\right) + \sum_{j=0}^{\infty} \beta^j E_t \mu_{t+j}.\end{aligned}$$

B The Patman Condition in Some Standard Cost Channel Models

Here we explore two simple models that are typical references for New Keynesian models with a cost channel, namely Ravenna and Walsh [2006] and the no-capital version of Rabanal [2007] proposed by Surico [2008]. Note that in those models, it is the nominal interest rate that enters the marginal cost and not the real interest rate. However, the T.E. Patman condition is computed holding expectations fixed, so that real and nominal rates move as one. Also note that the T.E. Patman condition is a necessary condition for inflation to increase following a rise in the interest rate when expectations are not held constant. If the T.E. Patman condition does not hold, then the G.E. Patman condition will not hold either, so that inflation will never respond positively to monetary tightening when the monetary shock is persistent.

B.1 Ravenna and Walsh [2006]

Firms must borrow the wage bill at the nominal interest rate. Preferences are $\frac{c^{1-\sigma}}{1-\sigma} - \chi\frac{N^{1+\eta}}{1+\eta}$. Euler equation and Phillips curve are given by:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}),$$
$$\pi_t = \beta E_t\pi_{t+1} + \kappa(\sigma+\eta)y_t + \kappa i_t.$$

The T.E. Patman condition writes $\frac{\gamma_r}{\gamma_y} > \alpha_r$, with $\gamma_r = \kappa$, $\gamma_y = \kappa(\sigma+\eta)$ and $\alpha_r = \frac{1}{\sigma}$. T.E. Patman condition implies $\frac{1}{\sigma+\eta} > \frac{1}{\sigma}$. It cannot hold as $\eta \geq 0$. Therefore, the T.E. Patman condition is never satisfied, and the G.E. Patman condition is not either.

B.2 Surico [2008]

Here, only a fraction θ of firms need to borrow the wage bill in advance. Euler equation and Phillips curve are given by:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}),$$
$$\pi_t = \beta E_t\pi_{t+1} + \kappa(\sigma+\eta)y_t + \kappa\theta i_t.$$

The T.E. Patman condition writes $\frac{\gamma_r}{\gamma_y} > \alpha_r$, with $\gamma_r = \theta\kappa$, $\gamma_y = \kappa(\sigma+\eta)$ and $\alpha_r = \frac{1}{\sigma}$. The T.E. Patman condition implies $\frac{1}{\sigma+\eta} > \frac{1}{\theta\sigma}$. A lower bound of the right-hand side is attained at $\theta = 1$. In that case, the T.E. Patman condition cannot hold as $\eta \geq 0$. This implies that the T.E. Patman condition cannot hold for values of θ lower than one. Therefore, the T.E. Patman condition is never satisfied, and the G.E. Patman condition is not either.

C Model Microfoundations

C.1 Discounted Euler Equation Specification

The derivation of the discounted Euler equation relies on two sets of assumptions. First, because of asymmetry of information and lack of commitment, individual households will face an upward sloping supply of funds when borrowing. To maintain tractability, we will consider an equilibrium in which agents never default, so that the income and wealth distributions will have a unique mass point. For exposition simplicity, we will derive the main features of the equilibrium in a two-period model and explain why the extension to an infinite horizon is trivial. Second, we will assume a particular timing of income and expenditure flows. Those two assumptions will allow us to derive a discounted Euler equation.

C.1.1 A simple two-period model with asymmetric information and lack of commitment

We consider a deterministic mode with two periods. There are two types of households and a zero-profit risk neutral representative bank that has access to an unlimited supply of funds at cost $\overline{R}$. Households receive no endowment in the first period, and ω in the second period. The consumption good is the numéraire.

Some households (superscript c) have access to commitment and always repay their debt while other households (superscript nc) cannot commit to repay. Type is not observable. Because of this, the risk neutral bank will want to charge a risk premium on its loans. More specifically, the bank proposes to the households a schedule $R(d)$ that is increasing in the level of debt d.

Preferences over consumption are given by $u(c_1) + \beta u(c_2)$. Households also bear an additively separable utility cost of defaulting $\psi(d)$ which is an increasing and convex function of the amount of defaulted debt.

When households borrow (as they will always do under regularity conditions on preferences u), they will consume (c_1, c_2) and their debt is $d = c_1$. Committed type households maximize their utility under the budget constraint $c_2 = \omega - R(c_1)c_1$. Their optimal choice for c_1 satisfies

$$u'(c_1^c) = \beta\Big(R(c_1^c) + R'(c_1^c)c_1^c\Big)u'(\omega - R(c_1^c)). \tag{C.1}$$

The non-committed type households optimally decide whether they will default (superscript d) or not (superscript nd) in period 2, and this choice can be made in period 1 because there is no uncertainty in this example. If they repay (no default), non-commited households behave as the committed type, so that

$$c_1^{nc,nd} = c_1^c.$$

If they default, then they will borrow (in period 1) as much as they need to equalise marginal utility of consumption with marginal psychological cost of default. The optimal choice will then satisfy:

$$u'(c_1^{nc,d}) = \psi'(c_1^{nc,d}), \tag{C.2}$$

while $c_2^{nc,d} = \omega$.

The optimal decision to default or not depends on the direction of the following inequality:

$$\underbrace{u(c_1^c) + \beta u\Big(\omega - R(c_1^c)c_1^c\Big)}_{\text{if no default}} \gtrless \underbrace{u(c_1^{nc-d}) + \beta u(\omega) - \psi(c_1^{nc,d})}_{\text{if default}}.$$

For given $u(\cdot)$, β and ω, there is always a psychological cost function $\psi(\cdot)$ such that household of the non-committed type choose to behave as committed households. In this case, we have a pooling equilibrium in which all households behave the same and in which there are no defaults. From the bank's zero-profit condition, we should have $R(c_1^c) = \overline{R}$ (as there is no default). This condition is the only restriction put on the $R(\cdot)$ schedule, so that any off-equilibrium belief $R'(\cdot) > 0$ is consistent with a no default pooling equilibrium.

Extension to an infinite horizon model : If we assume that past actions (default or not) are not observable, the logic of the two-period model still holds in a standard infinite horizon model. With asymmetric information on the household types (access or not to commitment), one can sustain an equilibrium with no default with the following properties: (i) households always make the same consumption and saving choices (no observed heterogeneity), (ii) there is no risk premium on the interest rate in equilibrium and (iii) households consistently face an upward sloping interest schedule $R(b)$. The interest of this modelling is the absence of observed heterogeneity that allows for a simple solving of the model.

C.1.2 Household's problem with upward sloping interest schedule.

There is a measure one of identical households indexed by i. Each household chooses a consumption stream and labor supply to maximizes discounted utility $E_0 \sum_{t=0}^{\infty} \beta^t \zeta_{t-1}(U(C_{it}) - \nu(L_{it}))$, where ζ is a discount shifter.

We split each period into a morning and an afternoon. There is no difference in information between morning and afternoon. In the morning, household i must order and pay consumption expenditures $P_t C_{it}$ and cannot use previous savings to do so. Household i must therefore borrow $D^M_{it+1} = P_t C_{it}$ units of money (say dollars) at a nominal interest rate i^H_{it} that, for the reasons mentioned above, will depend on her total borrowing in period t (hence the subscript i). In the afternoon, household i can borrow D^A_{it+1} for intertemporal smoothing motives, receives labor income $W_t L_{it}$ and profits from intermediate firms Ω_{it} and must repay principal and interest on the total debt inherited from the previous period $(1 + i^H_{it-1})(D^M_{it} + D^A_{it})$. The morning budget constraint is therefore given by:

$$D^M_{it+1} = P_t C_{it},$$

and the afternoon budget constraint writes:

$$D^A_{it+1} + W_t L_{it} + \Omega_{it} = (1 + i^H_{it-1})(D^M_{it} + D^A_{it}).$$

Putting these together, we obtain the following budget constraint for period t:

$$D^A_{it+1} + W_t L_{it} + \Omega_{it} = (1 + i^H_{it-1}) D^A_{it} + (1 + i^H_{it-1}) P_{t-1} C_{it-1}.$$

As there is no new information between morning and afternoon, the interest rate i^H_{it} faced by household i is a function of the total real net debt subscribed in period t. We write it as a premium over the risk-free nominal rate:

$$1 + i^H_{it} = (1 + i_t) \left(1 + \rho \left(\frac{D^M_{it+1} + D^A_{it+1}}{P_t}\right)\right) = (1 + i_t) \left(1 + \rho \left(C_{it} + \frac{D^A_{it+1}}{P_t}\right)\right),$$

with $\rho > 0$, $\rho' > 0$ and $\rho'' > 0$.

The decision problem of household i is therefore given by:

$$\max \sum_{t=0}^{\infty} \beta^t \zeta_{t-1} E_0 \left[U(C_{it}) - \nu(L_{it})\right],$$

$$\text{s.t.} \quad D^A_{it+1} + W_t L_{it} + \Omega_{it} = (1 + i^H_{it-1}) D^A_{it} + (1 + i^H_{it-1}) P_t C_{it},$$
$$1 + i^H_{it} = (1 + i_t) \left(1 + \rho \left(C_{it} + \frac{D^A_{it+1}}{P_t}\right)\right).$$

The first order conditions (evaluated at the symmetric equilibrium in which $D^A_{it+1} = 0$ $\forall i$) associated with this problem are:

$$U'(C_t) = \beta \frac{\zeta_t}{\zeta_{t-1}} E_t \left[U'(C_{t+1})(1 + i_t)\big(1 + \rho(C_t) + C_t \rho'(C_t)\big) \frac{P_t}{P_{t+1}}\right],$$
$$\frac{\nu'(L_t)}{U'(C_t)} = \frac{W_t}{P_t}.$$

Assuming that consumption utility is CRRA ($U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$), the Euler equation can be log-linearized to obtain (omitting constant terms and using $C_t = Y_t$) :

$$y_t = \alpha_y E_t[y_{t+1}] - \alpha_r (i_t - E_t[\pi_{t+1}]) + d_t,$$

where y is the log of Y and with $\alpha_y = \frac{\sigma}{\sigma+\varepsilon_\rho} \in]0, 1[$, $\alpha_r = \frac{1}{\sigma+\varepsilon_\rho} > 0$, $\varepsilon_\rho = \frac{C(2\rho'+C\rho'')}{\rho+C\rho'} > 0$ and $d_t = -\frac{1}{\sigma+\varepsilon_\rho}(\log \zeta_t - \log \zeta_{t-1})$. This give us equation (EE) in the main text.

C.1.3 Adding habit persistence

Assume that utility is $\frac{(C_{it}-\gamma C_{t-1})^{1-\sigma}}{1-\sigma} - \nu(L_{it})$. Note that we assume external habit. The first order conditions (evaluated at the symmetric equilibrium in which $D^A_{it+1} = 0$ $\forall i$) become:

$$(C_t - \gamma C_{t-1})^{-\sigma} = \beta \frac{\zeta_t}{\zeta_{t-1}} E_t \left[(C_{t+1} - \gamma C_t)^{-\sigma} (1 + i_t)\big(1 + \rho(C_t) + C_t \rho'(C_t)\big) \frac{P_t}{P_{t+1}}\right] \tag{C.3}$$
$$\frac{\nu'(L_t)}{(C_t - \gamma C_{t-1})^{-\sigma}} = \frac{W_t}{P_t}, \tag{C.4}$$

and the log-linearized Euler equation writes:

$$y_t = \alpha_{y,f} E_t[y_{t+1}] + \alpha_{y,b} y_{t-1} - \alpha_r (i_t - E_t[\pi_{t+1}]) + d_t.$$

C.2 Derivation of the Augmented New Keynesian Phillips Curve

The introduction of the real interest rate in the marginal cost of firms is not new (Christiano, Eichenbaum, and Evans [2005], Ravenna and Walsh [2006]). However, the twists we introduce here allow for arbitrary elasticities of the marginal cost with respect to respectively the real wage and the real interest rate. In what follows, we present the derivation of the marginal cost, that can be done considering the static optimal choice of inputs.

C.2.1 Production

Each monopolist produces a differentiated good using a basic input as the only factor of production, and according to a one to one technology. The marginal cost of production will therefore be the price of that basic input. It is assumed that the basic input is produced by a representative competitive firm. The representative firm produces basic input Q_t with labor L_t and the final good M_t according to the following Leontief technology:

$$Q_t = \min(a\Theta_t L_t, bM_t).$$

For implicity of the exposition, we assume that Θ_t is constant and normalized to one. The optimal production plan implies $Q_t = aL_t = bM_t$, so that the optimal input demands are $L_t = \frac{Q_t}{a}$ and $M_t = \frac{Q_t}{b}$. Denote by $\mathcal{C}(Q_t) = W_t L_t + \Phi_t M_t$ the total cost of production, where the exact expression of Φ_t will be derived later. Using the optimal input demands, we obtain:

$$\mathcal{C}(Q_t) = \left(\frac{W_t}{a} + \frac{\Phi_t}{b}\right) Q_t,$$

so that marginal cost is

$$\mathcal{C}'(Q_t) = \frac{W_t}{a} + \frac{\Phi_t}{b}.$$

Log-linearizing the above gives the following expression of the real marginal cost, where the variables are now in logs and where constant terms have been omitted:

$$mc_t = \left(\frac{\frac{W}{a}}{\frac{W}{a} + \frac{\Phi}{b}}\right)(w_t - p_t) + \left(\frac{\frac{\Phi}{b}}{\frac{W}{a} + \frac{\Phi}{b}}\right)(\phi_t - p_t).$$

C.2.2 Derivation of the cost Φ_t

The unit price of the final good that enters the production of basic input is P_t. We assume that, in the morning of each period, the basic input representative firm must borrow D^B_{t+1} at the risk-free nominal interest rate i_t to pay for the input M_t. In the afternoon, it produces, sells its production, pays wages, repays the debt contracted the previous period D^B_t and distributes all the profits Ω^B_t as dividends. Those profits will be zero in equilibrium. The period t budget constraint of the firm is therefore:

$$D^B_{t+1} + \widetilde{P}_t Q_t = W_t L_t + (1 + i_{t-1}) D^B_t + P_t M_t,$$

with $D^B_{t+1} = P_t M_t$. Period t profit writes:

$$\Omega^B_t = \widetilde{P}_t Q_t - W_t L_t - (1 + i_{t-1}) P_{t-1} M_{t-1},$$

where $\widetilde{P}_t$ is the price of the basic input. Assuming that the firm maximizes the expected discounted sum of profits real profits Ω^B_t / P_t with discount factor β, and using $Q_t = aL_t = bM_t$, we obtain the first order condition:

$$\widetilde{P}_t = \left(\frac{1}{a}\frac{W_t}{P_t} + \frac{\beta}{b} E_t\left[\frac{1 + i_t}{1 + \pi_{t+1}}\right]\right) P_t.$$

Therefore, the real marginal cost of the basic input firm will be given by:

$$MC_t = \frac{1}{a}\frac{W_t}{P_t} + \frac{\beta}{b} E_t\left[\frac{1 + i_t}{1 + \pi_{t+1}}\right].$$

Note that $\frac{1}{a}\frac{W_t}{P_t}$ can be expressed as $\frac{b}{b-1}\frac{W_t L_t}{P_t(Q_t-Y_t)}$, which is the labor share in total value added, so that a direct measure of the real marginal cost is

$$mc_t = \frac{b}{b-1} \times \text{labour share}_t + \frac{\beta}{b} E_t \left[\frac{1+i_t}{1+\pi_{t+1}} \right].$$

The price of the basic input $\widetilde{P}_t$ is equal to the nominal marginal cost of the basic input firm and is also equal to the marginal cost of the intermediate input firm (which is the relevant one for pricing decisions). In logs, the real marginal cost will write (omitting constants):

$$mc_t = \underbrace{\left(\frac{\frac{1}{a}\frac{W}{P}}{\frac{1}{a}\frac{W}{P} + \frac{\beta}{b}\frac{1+i}{1+\pi}} \right)}_{\widetilde{\gamma}_y} (w_t - p_t) + \underbrace{\left(\frac{\frac{\beta}{b}\frac{1+i}{1+\pi}}{\frac{1}{a}\frac{W}{P} + \frac{\beta}{b}\frac{1+i}{1+\pi}} \right)}_{\gamma_r} (i_t - E_t[\pi_{t+1}]).$$

C.2.3 Pricing

As in the standard New Keynesian model, intermediate firms play a Calvo lottery to draw price setting opportunities. Except for the use of the basic input, the modelling is very standard. The optimal household labor supply, that we will derive later, will give us:

$$\frac{\nu'(L_t)}{U'(C_t)} = \frac{W_t}{P_t},$$

which writes in logs, using $C_t = aL_t$ and omitting constant terms:

$$w_t - p_t = \left(\frac{L\nu''(L)}{\nu'(L)} - \frac{CU''(C)}{U'(C)} \right) y_t.$$

As $C_t = Y_t = aL_t$, the marginal cost does not depend on the scale of production and is the same for all the intermediate input firms. It is written as

$$mc_t = \underbrace{\widetilde{\gamma}_y \left(\frac{L\nu''(L)}{\nu'(L)} - \frac{CU''(C)}{U'(C)} \right)}_{\gamma_y} y_t + \gamma_r (i_t - E_t[\pi_{t+1}]).$$

The rest of the model is standard, and we obtain the New Keynesian Phillips curve:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \, mc_t + \mu_t.$$

Plugging in the expression for the real marginal cost, we have:

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa \Big(\gamma_y y_t + \gamma_r (i_t - E_t[\pi_{t+1}]) \Big) + \mu_t.$$

This give us equation (PC) in the main text.

C.2.4 Adding habit persistence

When habit persistence is added, labor supply depends on current and last period consumption (see section C.1.3). The Phillips curve writes :

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\Big(\gamma_y y_t + \gamma_{y,b} y_{t-1} + \gamma_r\left(i_t - E_t[\pi_{t+1}]\right)\Big) + \mu_t.$$

D Equivalence of Different Forms of Policy Rules

We show below that two classes of policy rules can replicate the same allocations. Those two classes are a standard Taylor rule that satisfies the Taylor principle:

$$i_t = \phi_y y_t + \phi_\pi \pi_t + \nu_t, \tag{D.1}$$

and a real interest rate rule:

$$i_t = E_t[\pi_{t+1}] + \psi_d d_t + \psi_\mu \mu_t + \psi_\nu \nu_t. \tag{D.2}$$

We prove the equivalence result in the fully forward New Keynesian model, but the proof can be easily extended to the model with a backward component.

The Euler equation and Phillips curve of the simple sticky prices model can be written as:[30]

$$X_t = AE_t[X_{t+1}] + B\Big(i_t - E_t[X_{t+1}]\Big) + CS_t, \tag{D.3}$$

where $X_t = (y_t, \pi_t)'$, $S_t = (d_t, \mu_t, \nu_t)'$ and each shock $x \in \{d, \mu, \nu\}$ follows $x_t = \rho_x x_{t-1} + \varepsilon_{xt}$. Denote R the diagonal matrix with the persistence parameters ρ_x on the diagonal, with $|\rho_x| < 1$. Let's also define $K = [0\ 1]$ so that $E_t[\pi_{t+1}] = KE_t[X_{t+1}]$.

Solution under a Taylor rule (D.1): Note that policy rule (D.1) can be written:

$$i_t = \Phi X_t + JS_t \tag{D.4}$$

with $\Phi = (\phi_y, \phi_\pi)$ and $J = [0\ 0\ 1]$. Plugging (D.4) in (D.3), we obtain:

$$X_t = \underbrace{(I - B\Phi)^{-1}(A - BK)}_{\mathcal{A}} E_t[X_{t+1}] + \underbrace{(I - B\Phi)^{-1}(BJ + C)}_{\mathcal{B}} S_t \tag{D.5}$$

We assume that the standard Taylor rule is restricted to give equilibrium determinacy, so that the eigenvalues of $\mathcal{A}$ are inside the unit disk.

Solving forward, we obtain :

$$X_t = \underbrace{\left(\sum_{i=0}^{\infty} \mathcal{A}^i \mathcal{B} R^i\right)}_{F(\Phi)} S_t.$$

Under the assumption that the equilibrium is determinate, $\sum_{i=0}^{\infty} \mathcal{A}^i \mathcal{B} R^i$ converges and $F(\Phi)$ is well defined.

[30] This does not cover the case where α_y is exactly 1. We can easily generalize the following analysis for this case.

Solution under the real interest rule (D.2): The policy rule (D.2) can be written:

$$i_t - E_t[\pi_{t+1}] = \underbrace{[\psi_d\ \psi_\mu \psi_\nu]}_{\Psi} S_t. \tag{D.6}$$

Plugging (D.6) in (D.3), we obtain:

$$X_t = AE_t[X_{t+1}] + \underbrace{(B\Psi + C)}_{\widehat{\mathcal{B}}} S_t. \tag{D.7}$$

Solving forward, we obtain:

$$X_t = \underbrace{\left(\sum_{i=0}^{\infty} A^i \widehat{\mathcal{B}} R^i\right)}_{\widehat{F}(\Psi)} S_t,$$

with $\Psi = (\psi_d, \psi_\mu, \psi_\nu)$. Ψ is uniquely defined given that A has its eigenvalues inside the unit disk as long as $|\alpha_y| < 1$.

Equivalence: Policy rules (D.1) and (D.2), which are respectively characterized by the parameters Φ and Ψ, will give similar allocations if:

$$F(\Phi) = \widehat{F}(\Psi).$$

Given a standard Taylor rule with parameters Φ that guarantees determinacy, the mapping $\widehat{F}$ is typically invertible. One can recover the equivalent real interest rule with parameters Ψ, that will be given by $\Psi = \widehat{F}^{-1}(F(\Phi))$.

E Data Definition and Sources

All series are final-vintage data.

Inflation : Headline CPI: Consumer Price Index for All Urban Consumers: All Items in U.S. City Average, Percent Change, Quarterly, Seasonally Adjusted, obtained from the FRED database, (`CPIAUCSL_PCH`). Sample is `1947Q1-2017Q3`.

Inflation : Consumer Price Index Retroactive Series, obtained from the BLS, U.S. city average, All items less food and energy, Monthly, Not Seasonally Adjusted, (`R-CPI-U-RS`). Sample is `1978M1-2020M12`

Domestic Producer Prices Index : Manufacturing for the United States, Change from Year Ago, Index 2015=100, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (`USAPPDMQINMEI_CH1`). Sample is `1961Q1-2021Q1`.

Expected Inflation : Expected Change in Price During the Next Year, obtained from the Surveys of Consumers, University of Michigan. Transformed into annualized quarterly expected inflation. Sample is `1960Q1-2017Q4`.

Expected Inflation : 10-year Expected Inflation, obtained from the Cleveland Fed. Sample is 1982M1-2021M9.

Nominal interest rate : Effective Federal Funds Rate, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (FEDFUNDS). Sample is 1954Q3-2017Q3.

Nominal interest rate : 3-Month Commercial Paper Rate, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (CP3M). Sample is 1971Q1-1997Q3.

Nominal interest rate : 3-Month AA Financial Commercial Paper Rate, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (CPF3M). Sample is 1907Q1-2021Q2.

Gross output : all industries, Millions of dollars, Annual, obtained from the BEA, Table TGO105-A. Sample is 1997-2020.

Intermediate Inputs : all industries, Millions of dollars, Annual, obtained from the BEA, Table TII105-A. Sample is 1997-2020.

Labour share : Nonfarm Business Sector, Index 2012=100, Quarterly, Seasonally Adjusted, obtained from the FRED database, (PRS85006173). Sample is 1947Q1-2021Q1.

Unemployment : Civilian Unemployment Rate, Percent, Quarterly, Seasonally Adjusted, obtained from the FRED database, (UNRATE). Sample is 1948Q1-2017Q3.

Unemployment : Noncyclical Rate of Unemployment, Percent, Quarterly, Not Seasonally Adjusted, obtained from the FRED database, (NROU). Sample is 1949Q1-2017Q3.

Unemployment gap : constructed as UNRATE - NROU.

Monetary Shocks : obtained from Wieland and Yang [2020] who have followed the method in Romer and Romer [2004] on an extended sample. Sample is 1969Q1-2007Q4.

International nominal interest rates : The measure of the nominal interest rate is either the "Immediate interest rates, Call Money, Interbank Rate" or the "Short-term interest rates" depending on availability . Data are taken from the Oecd MEI database.

F Transforming Year-to-Year Inflation Expectations into Quarter-to-Quarter Ones

In the Michigan Survey of Consumers, every month a representative sample of consumers are asked the following question: "*By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*" The answer to this question is then the one-year-ahead inflation expectation $E_t\pi_{t+4,t}$. To keep consistency with the quarter-to-quarter inflation we use in the estimation, we rescale the one-year-ahead expected inflation in the following way.[31]

We first assume that realized quarter-to-quarter inflation follows an AR(1) process with persistence ρ_π:

$$\pi_{t+1,t} = \rho_\pi \pi_{t,t-1} + \epsilon_t \tag{F.1}$$

Consumers may or may not have the correct belief on ρ_π. We assume they believe that persistence is $\widetilde{\rho}$, so that the perceived law of motion of inflation is

$$\pi_{t+1,t} = \widetilde{\rho}\pi_{t,t-1} + \epsilon_t \tag{F.2}$$

Consumers observe a noisy signal on inflation: $s_t = \pi_{t,t-1} + \eta_t$ where η_t is of mean zero, i.i.d., orthogonal to ϵ_t and independent across time. Consumers will form quarter-to-quarter inflation expectation, denoted by $E_t\pi_{t+1,t}$, using a Kalman filter:

$$E_t\pi_{t+1,t} = \widetilde{\rho}E_t\pi_{t,t-1} = \widetilde{\rho}(1-K)E_{t-1}\pi_{t,t-1} + \widetilde{\rho}K\pi_{t,t-1} + \widetilde{\rho}K\eta_t \tag{F.3}$$

where K is the Kalman gain.

We do observe one-year-ahead expected inflation:

$$E_t\pi_{t+4,t} \equiv E_t(\pi_{t+4,t+3} + \pi_{t+3,t+2} + \pi_{t+2,t+1} + \pi_{t+1,t})$$

Using the perceived law of motion (F.2):

$$\begin{aligned} E_t\pi_{t+4,t} &= (1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)E_t\pi_{t+1,t} \\ &= (1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)\big(\widetilde{\rho}(1-K)E_{t-1}\pi_{t,t-1} + \widetilde{\rho}K\pi_{t,t-1} + \widetilde{\rho}K\eta_t\big) \end{aligned} \tag{F.4}$$

We use the $t-1$ version of (F.4) and plug it in the above equation to obtain:

$$\begin{aligned} E_t\pi_{t+4,t} = &\underbrace{\widetilde{\rho}(1-K)}_{\psi_1} E_{t-1}\pi_{t+3,t-1} + \underbrace{(1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)\widetilde{\rho}K}_{\psi_2}\pi_{t,t-1} \\ &+ (1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3)\widetilde{\rho}K\eta_t \end{aligned} \tag{F.5}$$

We can estimate equation (F.5) with OLS because η_t is the i.i.d noise orthogonal to inflation. We need to use quarter-to-quarter (not annualized) inflation for $\pi_{t,t-1}$ and year-ahead expected inflation and its lag from the Michigan Survey of Consumers. We consider Headline CPI as proxy for $\pi_{t,t-1}$ here, but the implied estimates for $\tilde{\rho}$ are very close to those obtained

Table F.1: Estimation of Equation (F.5)

OLS for:	$E_t\pi_{t+4,t} = \psi_1 E_{t-1}\pi_{t+3,t-1} + \psi_2\pi_{t,t-1} + \psi_2\eta_t$	
Sample:	1969-2007	1978-2007
ψ_1	0.45	0.64
	(0.104)	(0.046)
ψ_2	1.53	1.02
	(0.277)	(0.138)
Implied persistence:		
$\widetilde{\rho}$	0.89	0.93

Notes: The constant term is omitted from the table. Newey-West standard errors reported in brackets. Measure of inflation in use is Headline CPI.

using Core CPI. We first use sample from 1969-2007 to guarantee it lines up with the sample in Table 1 and Table 2, and we use sample from 1978-2007 for Table 3.

Given the estimate on the perceived persistence of inflation, the quarter-to-quarter expected inflation is implied by equation (F.4):

$$E_t\pi_{t+1,t} = \frac{1}{1+\widetilde{\rho}+\widetilde{\rho}^2+\widetilde{\rho}^3} E_t\pi_{t+4,t} \tag{F.6}$$

G Estimating the Phillips Curve Using Year-to-Year Inflation

We start by deriving a version of Equation (8) in the main text using four-quarter inflation, that we denote $\pi_{t,t-4}$. For periods $t, t-1, t-2$ and $t-3$, Equation (8) writes

$$\begin{aligned} \pi_{t-j,t-j-1} &= \beta E_{t-j}\pi_{t-j+1,t-j} + \gamma_y x_{t-j} + \gamma_r(i_{t-j} - E_{t-j}\pi_{t-j+1,t-j}) + \mu_{t-j} \\ &= (\beta-\gamma_r)E_{t-j}\pi_{t-j+1,t-j} + \gamma_y x_{t-j} + \gamma_r i_{t-j} + \mu_{t-j} \quad \forall j = 0,1,2,3. \end{aligned} \tag{G.1}$$

Summing the above equation up for all $j = 0, 1, 2, 3$, we obtain

$$\begin{aligned} \pi_{t,t-4} =& \pi_{t,t-1} + \pi_{t-1,t-2} + \pi_{t-2,t-3} + \pi_{t-3,t-4} \\ =& (\beta-\gamma_r)(E_t\pi_{t+1,t} + E_{t-1}\pi_{t,t-1} + E_{t-2}\pi_{t-1,t-2} + E_{t-3}\pi_{t-2,t-3}) \\ &+ \gamma_y(x_t + x_{t-1} + x_{t-2} + x_{t-3}) + \gamma_r(i_t + i_{t-1} + i_{t-2} + i_{t-3}) \\ &+ \mu_t + \mu_{t-1} + \mu_{t-2} + \mu_{t-3}. \end{aligned} \tag{G.2}$$

Taking expectation at time $t-3$ and applying the law of iterated expectation, we obtain

$$\begin{aligned} E_{t-3}(\pi_{t,t-4}) =& (\beta-\gamma_r)E_{t-3}\pi_{t+1,t-3} + \gamma_y E_{t-3}(x_t + x_{t-1} + x_{t-2} + x_{t-3}) \\ &+ \gamma_r E_{t-3}(i_t + i_{t-1} + i_{t-2} + i_{t-3}) + \mu_{t-3} \end{aligned} \tag{G.3}$$

[31] For details of this approach extended to multi-variable joint learning environment, see Hou [2020].

Notice that $\pi_{t,t-4}$ contains information (about shocks) from $t-3$ up to t. Adding $\pi_{t,t-4}$ and subtracting $E_{t-3}(\pi_{t,t-4})$ from both sides, we obtain

$$\begin{aligned}\pi_{t,t-4} =&(\beta - \gamma_r)E_{t-3}\pi_{t+1,t-3} + \gamma_y E_{t-3}(x_t + x_{t-1} + x_{t-2} + x_{t-3}) \\ &+ \gamma_r E_{t-3}(i_t + i_{t-1} + i_{t-2} + i_{t-3}) + \mu_{t-3} + \underbrace{(\pi_{t,t-4} - E_{t-3}\pi_{t,t-4})}_{\epsilon_{t,t-3}}\end{aligned} \tag{G.4}$$

In the above equation, the term $\epsilon_{t,t-3}$ contains shocks realized after $t-3$, including the monetary shocks. Denote $I_{t,t-3} = i_t + i_{t-1} + i_{t-2} + i_{t-3}$ and $X_{t,t-3} = x_t + x_{t-1} + x_{t-2} + x_{t-3}$ to simplify notations. We add and subtract $I_{t,t-3}$ and $X_{t,t-3}$ to the right hand side of (G.4) to obtain:

$$\begin{aligned}\pi_{t,t-4} =&\beta E_{t-3}\pi_{t+1,t-3} + \gamma_y X_{t,t-3} + \gamma_r(I_{t,t-3} - E_{t-3}\pi_{t+1,t-3}) \\ &+ \underbrace{\mu_{t-3} + \epsilon_{t,t-3} - \gamma_y(X_{t,t-3} - E_{t-3}X_{t,t-3}) - \gamma_r(I_{t,t-3} - E_{t-3}I_{t,t-3})}_{\omega_{t,t-3}}\end{aligned} \tag{G.5}$$

Now notice the error term $\omega_{t,t-3}$ include time $t-3$ cost-push shock μ_{t-3}, and any shocks happening from time $t-3$ to t. To estimate β, γ_y and γ_r, we need to instrument with monetary shocks at time $t-3$ and in earlier periods. Monetary policy shocks at $t-3$ and earlier are indeed valid instruments because they are orthogonal to cost-push shocks at $t-3$ and to any realized shocks between $t-3$ and t.

H Estimating an Hybrid Phillips Curve

Table H.1 shows estimates of a "hybrid" version of the Phillips curve of the type:

$$\pi_t = \beta_f \pi^e_{t+1} + \beta_b \pi_{t-1} + \gamma_y x_t + \gamma_r(i_t - \pi^e_{t+1}) + \mu_t, \tag{H.1}$$

We find again an insignificant slope γ_y and a positive and significant at 1% cost channel parameter γ_r.

I More Details on the Full Informatiom Estimations

I.1 The Simple Model with Constrained Value for α_r

We impose the value of output elasticity to the real interest rate α_r to be equal to the point estimate obtained by Smets and Wouters [2007] (α_r =1/1.39) and re-estimate the simple New-Keynesian model by Maximum Likelihood. As shown in Table I.1, results are qualitatively unaffected: the slope of the Phillips curve is negative but small, the cost channel γ_r is positive and significant. The Patman condition is satisfied and the response of inflation to a monetary shock is positive and significant, although very small.

Table H.1: Estimation of the Hybrid Phillips Curve

π	Headline CPI		Core R-CPI	
	(1)	(2)	(3)	(4)
β_f	0.56	0.67	0.51	0.44
	(0.095)	(0.103)	(0.059)	(0.036)
β_b	0.49	0.44	0.46	0.54
	(0.058)	(0.075)	(0.067)	(0.041)
γ_y	0.04	-0.04	0.01	0.0138†
	(0.042)	(0.060)	(0.005)	(–)
γ_r		0.13	0.03	0.02
		(0.048)	(0.009)	(0.008)
Observations	150	150	118	118
J Test	3.585	4.765	10.069	8.123
(jp)	(0.981)	(0.906)	(0.434)	(0.702)
Weak ID Test	1.495	1.473	7.237	61.945

Notes: All results are using IV-GMM procedure, Newey-West HAC standard errors with six lags are reported in parentheses. The constant term is omitted from the table. The measure of inflation is BLS "Consumer Price Index retroactive series using current methods for all items less food and energy", the measure of market tightness is the U.S. Congressional Budget Office unemployment gap. We use the Michigan Survey of Consumers to measure inflation expectations is the MSC columns, and assume Full Information Rational Expectations in the FIRE ones. Real oil price is added as a control in all the equations and all regressors are instrumented using six lags of Romer and Romer [2004] shocks (as extended by Wieland and Yang [2020]) and their squares as instruments. For γ_y and γ_r, estimates highlighted in grey are significant at 1% and not significant at 10% if not highlighted. Sample is 1969Q1-2007Q4.

Table I.1: Estimated Parameters, Simple Model, Imposing α_r =1/1.39

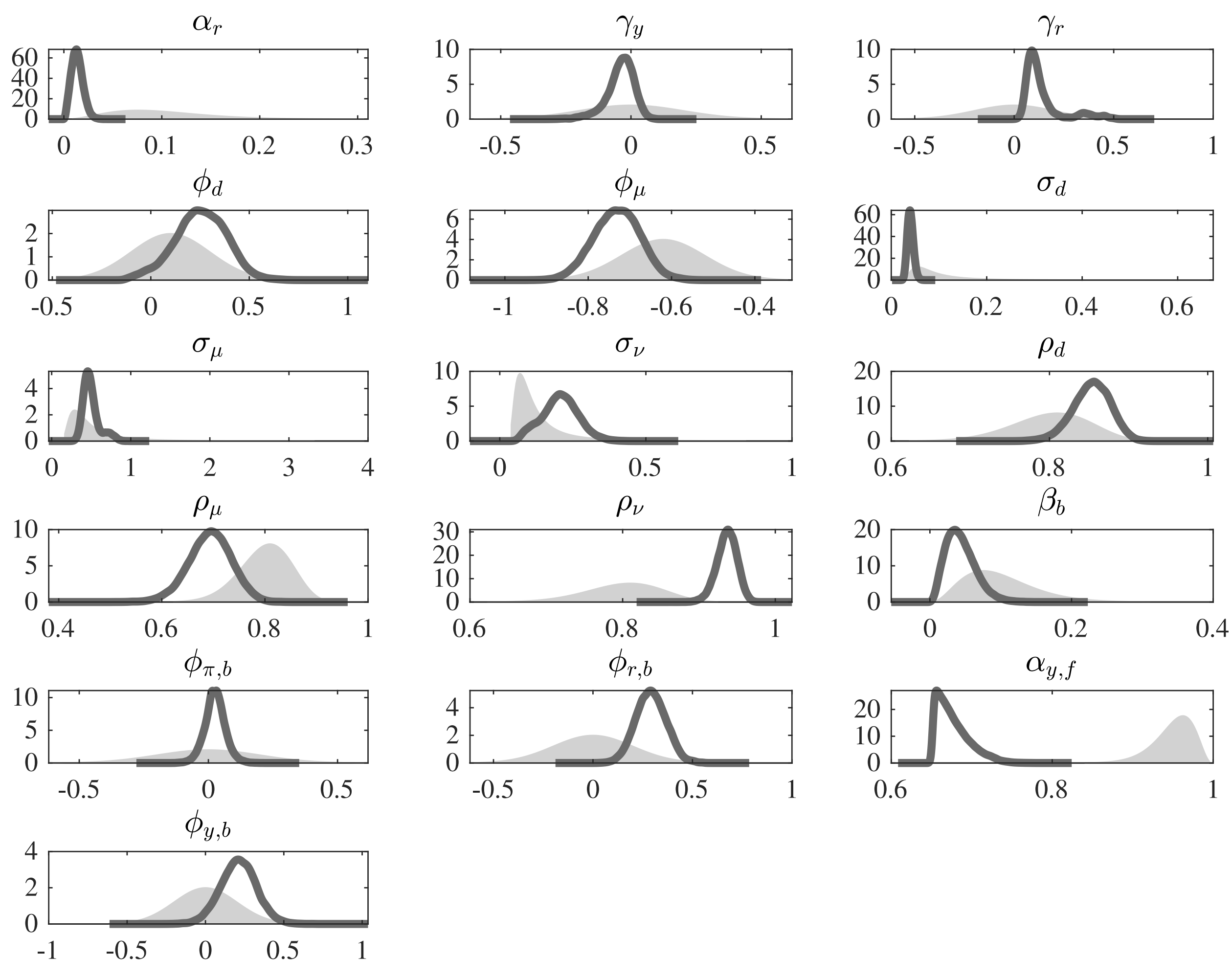
α_r	0.72†	γ_y	-0.02	γ_r	0.05			
	(–)		(0.01)		(0.02)			
ϕ_d	0.62	ϕ_μ	-0.07	σ_d	0.46	σ_μ	0.89	
	(0.09)		(0.03)		(0.07)		(0.10)	
σ_ν	0.01	ρ_d	0.95	ρ_μ	0.10	ρ_ν	0.98	
	(0.01)		(0.04)		(0.08)		(0.02)	
T.E. Patman condition							0.059	(0.027)
G.E. Patman condition							0.001	(0.000)

Notes: this table shows the estimated coefficients of equations (EE), (PC) and (Policy) with unemployment gap, Core CPI Research Series. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. Standard errors are between parenthesis, † denotes a parameter value that is imposed and not estimated. Sample runs from 1978Q2 to 2007Q4. T.E. Patman condition corresponds to $\gamma_r - \alpha_r \gamma_y$, G.E. Patman condition is the impact response of inflation π to a one standard deviation monetary policy shock.

I.2 The Extended Model in the Baseline Case

We assume relatively dispersed priors. For the parameters that were estimated in the simple model, we center the prior distributions on the previously estimated value. For the new parameters, we center the priors around zero. Figure I.1 displays prior and posterior distributions for all the estimated parameters. One can check that all the parameters are indeed well identified. Table I.2 presents mode details about the prior and posterior distributions.

Figure I.1: Prior and Estimated Posterior Distributions for Parameters, Extended Model, Baseline



Notes: this figure plots the prior (the light gray area) and posterior (the dark gray line) distributions for the extended model parameters. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chain.

Table I.2: Detailed Results on Parameters Estimation, Extended Model, Baseline

	Prior distribution			Max. posterior		Posterior distribution MH			
Parameter	Type	a	b	Mode	s.d. (Hessian)	Mean	Med.	2.5%	97.5%
α_r: Euler coef. on real rate	Beta([a,b])	0.10	0.05	0.02	0.01	0.02	0.02	0.01	0.03
γ_y: Marginal cost loading to labour market	Normal([a,b])	0.00	0.20	-0.04	0.04	-0.03	-0.03	-0.11	0.04
γ_r: Marginal cost loading to the real interest rate	Normal([a,b])	0.00	0.20	0.06	0.02	0.07	0.06	0.03	0.11
ϕ_d: Policy rule reaction to demand shock	Normal(a,b)	0.10	0.20	0.53	0.11	0.51	0.51	0.30	0.73
ϕ_μ: Policy rule reaction to markup shock	Normal(a,b)	-0.62	0.10	-0.70	0.07	-0.71	-0.71	-0.85	-0.58
σ_d: Demand shock s.d.	InvGamma(a,b)	0.12	2.00	0.04	0.01	0.04	0.04	0.03	0.06
σ_μ: Markup shock s.d	InvGamma(a,b)	0.61	2.00	0.36	0.05	0.38	0.38	0.29	0.48
σ_ν: Monetary shock s.d.	InvGamma(a,b)	0.15	2.00	0.23	0.07	0.28	0.28	0.17	0.43
ρ_d: Demand shock persistence	Beta([a,b])	0.80	0.05	0.86	0.02	0.85	0.85	0.79	0.90
ρ_μ: Markup shock persistence	Beta([a,b])	0.80	0.05	0.62	0.05	0.62	0.62	0.52	0.72
ρ_ν: Monetary shock persistence	Beta([a,b])	0.80	0.05	0.94	0.01	0.94	0.94	0.91	0.96
β_b: Phillips curve intertia	Beta(a,b)	0.10	0.05	0.04	0.02	0.05	0.05	0.01	0.11
$\phi_{\pi,b}$: Past inflation in policy rule	Normal(a,b)	0.00	0.20	0.07	0.06	0.01	0.02	-0.13	0.14
$\phi_{r,b}$: Persistence in policy rule	Normal(a,b)	0.00	0.20	0.18	0.08	0.14	0.14	-0.01	0.28
$\alpha_{y,f}$: Habit persistence	Beta(a,b)	0.95	0.03	0.73	0.04	0.75	0.74	0.66	0.88
$\phi_{y,b}$: Past gap in policy rule	Normal(a,b)	0.00	0.20	0.04	0.18	0.09	0.09	-0.23	0.42

Notes: this table shows the estimated coefficients of equations (EE'), (PC") and (Policy') using unemployment gap, Core R-CPI and the sample is 1978Q2-2007Q4. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chains. "Med." is the median of the posterior distribution.

I.3 Estimating with Phillips Curve (PC') Instead of (PC")

Here we repeat the benchmark estimation but we use Phillips curve (PC')

$$\pi_t = \beta\big((1-\beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}\big) + \kappa\big(\gamma_y y_t + \gamma_{y,b} y_{t-1} + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t, \tag{PC'}$$

instead of (PC").

$$\pi_t = \beta\big((1-\beta_b)E_t[\pi_{t+1}] + \beta_b\pi_{t-1}\big) + \kappa\big(\gamma_y y_t + \gamma_r(i_t - E_t[\pi_{t+1}])\big) + \mu_t. \tag{PC"}$$

Table I.3 shows that all the parameters are close to what was estimated in the benchmark case and the Patman condition is again satisfied. Table I.4 gives more details about the prior and posterior distributions.

Table I.3: Estimated Parameters, Extended Model with Phillips Curve (PC')

α_r	0.02	γ_y	-0.07	γ_r	0.07	ϕ_d	0.51
	[0.01, 0.03]		[-0.26, 0.12]		[0.03, 0.11]		[0.31, 0.73]
ϕ_μ	-0.71	σ_d	0.04	σ_μ	0.38	σ_ν	0.28
	[-0.85, -0.59]		[0.03, 0.06]		[0.28, 0.47]		[0.17, 0.44]
ρ_d	0.85	ρ_μ	0.62	ρ_ν	0.94	β_b	0.05
	[0.79, 0.90]		[0.52, 0.72]		[0.91, 0.96]		[0.01, 0.10]
$\phi_{\pi,b}$	0.01	$\phi_{r,b}$	0.14	$\alpha_{y,f}$	0.75	$\phi_{y,b}$	0.09
	[-0.13, 0.15]		[-0.02, 0.29]		[0.66, 0.88]		[-0.22, 0.43]
$\gamma_{y,b}$	0.04						
	[-0.13, 0.21]						
T.E. Patman condition			0.07	[0.03, 0.11]			
G.E. Patman condition			0.10	[0.07, 0.13]			

Notes: this table shows the posterior median estimates of the coefficients in equations (EE'), (PC') and (Policy') using unemployment gap, Core CPI and the sample is 1978Q2-2007Q4. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chain. The numbers between brackets represent the 90% confidence band using the posterior distribution. T.E. Patman condition corresponds to $\gamma_r - \alpha_r\gamma_y$, G.E. Patman condition is the impact response of inflation π to a one standard deviation monetary policy shock.

Table I.4: Detailed Results on Parameters Estimation, Extended Model with Phillips Curve (PC')

Parameter	Prior distribution			Max. posterior		Posterior distribution MH			
	Type	a	b	Mode	s.d. (Hessian)	Mean	Med.	2.5%	97.5%
α_r: Euler coef. on real rate	Beta([a,b])	0.10	0.05	0.02	0.01	0.02	0.02	0.01	0.03
γ_y: Marginal cost loading to labour market	Normal([a,b])	0.00	0.20	-0.08	0.10	-0.07	-0.07	-0.26	0.12
γ_r: Marginal cost loading to the real interest rate	Normal([a,b])	0.00	0.20	0.06	0.02	0.07	0.06	0.03	0.11
ϕ_d: Policy rule reaction to demand shock	Normal(a,b)	0.10	0.20	0.53	0.15	0.51	0.51	0.31	0.73
ϕ_μ: Policy rule reaction to markup shock	Normal(a,b)	-0.62	0.10	-0.70	0.08	-0.71	-0.71	-0.85	-0.59
σ_d: Demand shock s.d.	InvGamma(a,b)	0.12	2.00	0.04	0.02	0.04	0.04	0.03	0.06
σ_μ: Markup shock s.d	InvGamma(a,b)	0.61	2.00	0.36	0.05	0.38	0.38	0.28	0.47
σ_ν: Monetary shock s.d.	InvGamma(a,b)	0.15	2.00	0.23	0.08	0.28	0.28	0.17	0.44
ρ_d: Demand shock persistence	Beta([a,b])	0.80	0.05	0.85	0.03	0.85	0.85	0.79	0.90
ρ_μ: Markup shock persistence	Beta([a,b])	0.80	0.05	0.62	0.05	0.62	0.62	0.52	0.72
ρ_ν: Monetary shock persistence	Beta([a,b])	0.80	0.05	0.94	0.01	0.94	0.94	0.91	0.96
β_b: Phillips curve intertia	Beta(a,b)	0.10	0.05	0.04	0.02	0.05	0.05	0.01	0.10
$\phi_{\pi,b}$: Past inflation in policy rule	Normal(a,b)	0.00	0.20	0.07	0.07	0.01	0.01	-0.13	0.15
$\phi_{r,b}$: Persistence in policy rule	Normal(a,b)	0.00	0.20	0.18	0.07	0.14	0.14	-0.02	0.29
$\alpha_{y,f}$: Habit persistence	Beta(a,b)	0.95	0.03	0.72	0.06	0.75	0.74	0.66	0.88
$\phi_{y,b}$: Past gap in policy rule	Normal(a,b)	0.00	0.20	0.04	0.15	0.09	0.09	-0.22	0.43
$\gamma_{y,b}$: Marginal cost loading to past labour market	Normal(a,b)	0.00	0.10	0.04	0.09	0.04	0.04	-0.13	0.21

Notes: This table shows the estimated coefficients of equations (EE'), (PC') and (Policy') using unemployment gap, Core CPI and the sample 1978Q2–2007Q4. Parameters β and α_y are not estimated and set to .99 and .99. Parameter κ is normalized to one. The posterior distribution is obtained using the Random Walk Metropolis Algorithm, with two chains of 1,000,000 draws each and discarding the first 500,000 draws of each chain. "Med." is the median of the posterior distribution.