Convergence Across Castes*

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Abstract

India witnessed a sharp education and wage catch-up by the historically disadvantaged scheduled castes and tribes (SC/STs) towards non-SC/ST levels during the period 1983-2012. We provide a structural explanation for the catch-up using a multi-sector, heterogenous agent model where individuals differ in ability and their caste identity. Castes differ in the costs of schooling and accessing sectoral labor markets which results in caste-based talent misallocations. We show that exogenous productivity growth can explain 72 percent of the observed wage convergence. The primary driver of convergence in the model is the fall in real costs of schooling with growth. We provide independent evidence in support of this mechanism.

JEL Classification: J6, R2

Keywords: Castes, convergence, growth

1 Introduction

Caste identities are fundamentally enmeshed in the social, economic and political lives of people in India. These identities, which are determined by birth, often dictate where they grow up, where they study, where they work, who they marry and who they network with. Given the rigid nature

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of birth-based caste hierarchies, it was often viewed as an immovable barrier to socio-economic mobility. With the original caste identities being based on occupations, the lack of occupational mobility turned some of the castes at the lower end of the occupation hierarchy into permanent under-classes in Indian society.[1]

After India's independence from British rule in 1947, framers of the constitution of modern India were acutely aware of the need to create a level playing field for all Indians in the newly independent country. To facilitate this, the constitution identified the most disadvantaged castes and tribes and collected them in a schedule of the Indian constitution. This identified group is today referred to as Scheduled Castes and Tribes or SC/STs. The constitution mandated reservations for SC/STs in higher education, public sector jobs and political representation as a way of redressing their historical disadvantage.

Recent work by Hnatkovska et al. (2012) has shown that the period since 1983 has witnessed a sharp reduction in education, occupation and wage disparities between non-SC/STs and SC/STs with most of the wage convergence being empirically accounted for by education convergence.[2] These trends represent a remarkable turnaround in the fortunes of SC/STs after centuries of disadvantage.

What sparked this convergence? There are no clear answers to this question even though answers are crucial from a policy standpoint. Munshi (2019) conjectures that affirmative action policies as well as caste-based labor networks may have been contributing factors. While there is scholarly work that examines the effects of networks and reservations in local contexts, there is no evidence on the dynamic effects of either of these caste-specific mechanisms on caste gaps at the aggregate level.[3]

Despite the lack of systematic evidence on their aggregate effects, the demand for more caste based social welfare and economic protections has continued to rise in India. In 2022, the state of Bihar conducted a survey to enumerate the caste distribution at a granular level that went beyond the coarse SC/ST-non-SC/ST binary classification. The goal of this effort is for the state government to better target social welfare schemes to different castes. Many other states in India are

[1] Munshi (2019) provides an excellent background and overview of the role of castes in India.

[2] On a related theme, Hnatkovska et al. (2013) show that the intergenerational mobility rates of SC/STs in terms of education, occupation choices and wages have also caught up to non-SC/ST rates.

[3] Munshi and Rosenzweig (2016) shows evidence for the effects of caste networks on migration decisions in India. Their results suggest that caste networks may have slowed down urban migration. On the effects of reservations, Bertrand et al. (2010) examines their impact on enrolment and outcomes in engineering colleges in one Indian state in 1996. However, neither paper nor the mechanisms that they examine map directly into the time series evolution of aggregate caste wage gaps, which is the focus of this paper.

also considering conducting similar surveys. The implicit consensus appears to be that inequalities across castes are best addressed by targeted social welfare and affirmative action programs.

There are reasons, however, to believe that factors other than caste-specific policies or mechanisms may have been at play in India during this period. Specifically, other key measures of inter-group inequality in India such as rural-urban gaps (Hnatkovska et al 2022) and gender gaps (Hnatkovska et al 2016) have also declined sharply since 1983. These trends suggest that important non-caste specific factors may also have been at play.

This paper examines the role of one such non-caste specific factor: the role of aggregate macroeconomic growth. The proximate reason for focusing on growth is the well-known economic liberalization in India in 1991 which sparked a big economic turnaround in a previously moribund economy. Aggregate growth rates rose from an anemic annual average of 3-3.5 percent before 1991 to close to 6 percent in the period since. This growth pick-up was accompanied by an intensifying structural transformation of the economy from agrarian to non-agrarian.

There are important policy implications of this assessment of the role of growth. If growth and the accompanying structural transformation of the economy accounts for a significant share of the caste convergence, the role of caste-based welfare and affirmative action policies in helping disadvantaged castes catch up becomes much diminished. Put differently, if growth tends to lift all boats, then the policy focus is better aimed at enhancing aggregate growth rather than on group-specific interventions.

For sure, this period also witnessed other policy initiatives which may have had an independent bearing on caste gaps. Initiatives such as Operation Blackboard (1986) and the Right to Education Act (2002) aimed to make education available more widely. The local governance reform of 1992 decentralized governance to village councils called Gram Panchayats while also providing women and SC/STs reservations on these councils. These initiatives may have facilitated better provisioning of public goods like schools which, in turn, could have benefited SC/STs. Our results allows one to potentially attribute the overall caste convergence that was induced by such caste-based policy initiatives as the part that is left *unexplained* by growth.

We conduct our study by developing a three-sector model (agriculture, manufacturing and services) of an economy with heterogenous agents. Agents differ along two dimensions. First, agents are different in their innate ability endowment which they all draw from a common ability distribution. Second, agents in the model belong to one of two castes: non-SC/STs and SC/STs.

Castes in the model differ on two dimensions: (a) the cost of acquiring schooling; and (b) the cost of accessing sectoral labor markets. These cost differences imply that even though the ability distributions of individuals in the two castes are identical, there is a misallocation of ability which generates caste gaps in sectoral employment and wages. Overall sectoral labor productivity depends both on exogenous productivity as well as worker allocations in education and sectors. Consequently, talent misallocations in the model reduce equilibrium labor productivity while improvements in allocations improve productivity.

We use the model to quantitatively assess the effect of changes in sectoral productivities on schooling and sectoral misallocations during 1983-2012. Specifically, we examine the explanatory power of these productivity changes for the observed decline in the sectoral caste employment gaps, the sectoral caste wage gaps as well as the overall caste wage convergence observed in the data.

The key parameters of the model, including the caste-specific costs of schooling and sectoral labor market access, are calibrated to match the 1983 levels of the sectoral caste employment gaps, sectoral caste wage gaps and the average education levels of the two castes. Our baseline calibration identifies higher schooling costs for SC/STs as the primary cause of the large sectoral caste gaps in employment and wages in 1983.[4]

Armed with the calibrated model for 1983, we conduct a sequence of quantitative experiments to examine the importance of productivity growth. Our experiments yield four key results. First, exogenous sectoral productivity growth during 1983-2012 induces a decrease in the caste wage gap in the model that is 72 percent of the decline in the data.

Second, we find that a faster increase in the education attainment rates of SC/STs accounts for most of the wage convergence in the model. Intuitively, real costs of schooling decline with aggregate growth. Since SC/STs start with higher costs of schooling in the model, their schooling costs fall proportionately faster with growth. This sparks the relatively faster increase in SC/ST schooling and wages in the model.

We provide two independent pieces of evidence in support of this mechanism. First, we use panel data on states in India for 1983-2012 to show that the cost of schooling (proxied by teacher salaries) rises less than proportionately with per capita incomes. This result echoes a similar finding in the cross-country data reported by Banerjee and Duflo (2005).[5] This provides direct evidence

[4]Higher education costs for SC/STs might seem counterintuitive to the reader since India has had affirmative action programs for education since 1952. These programs provide reserved seats for SC/STs in colleges and universities. Education costs however depend on much more basic things like provisioning of primary, middle and secondary schools. We show below evidence of systematic under-provisioning of schools in SC/ST dominated geographical units, which provides support for SC/STs facing higher schooling costs despite the affirmative action programs.

[5]Teacher salaries represent the largest component of education spending, comprising 80-90 percent of education

for declining real costs of education with growth, which is the key driver of education convergence in the model. Second, using census data we show that while initial school provisioning was lower in SC/STs dominated villages in India in 1991, school provisioning increased relatively faster in SC/STs dominated villages during 1991-2011. This is suggestive evidence of a faster reduction in schooling costs for SC/STs.

Third, sectoral labor productivity in the model depends on exogenous productivity as well as the endogenous sorting of workers in schooling and sectoral employment. The caste distortion creates a misallocation of talent and reduces overall labor productivity. Our quantitative exercise finds that the endogenous reduction in caste-based talent misallocation in response to exogenous productivity growth accounts for 45 percent, 37 percent and 11 percent of the overall sectoral labor productivity growth in Agriculture, Manufacturing and Services, respectively. This amounts to 39 percent of the aggregate share-weighted labor productivity growth in the India during 1983-2012.

Fourth, the model allows us to compute the welfare costs of caste distortions. We do this by equalizing both schooling costs and sectoral entry costs across castes. Equalizing all schooling and sectoral entry costs across castes increases average per capita consumption by 10.2 percent in 1983 and 10.3 percent in 2012. Correspondingly, removing all caste distortions results in per capita output rising by 11.4 percent in 1983 and 8.5 percent in 2012. The gains for SC/STS are obviously larger than these overall numbers.

We interpret our results as suggesting that the rapid growth take-off in India over the past three decades has induced a dramatic narrowing of the historical economic disparities faced by SC/STs. The primary driver of this convergence has been the relatively faster increase in the education attainment rates of SC/STs. While other caste-specific policies may have well played a role as well, growth did most of the work by lifting all boats.[6]

The paper is related to three distinct bodies of work. The first is the work on castes in India and their impact on economic outcomes. Aside from the contributions of Hnatkovska et al. (2012). and Hnatkovska et al. (2013) cited above, notable other contributors to this literature are Banerjee and Knight (1985), and Borooah (2005) who examined the discrimination against SC/STs in labor

budgets in advanced countries.

[6]Our work also evaluates the relative importance of two other features of the Indian economy. We assess the importance of job reservations for SC/STs in India. Our results suggest that this affirmative action policy may have lowered the ***levels*** of the caste wage gaps but likely did not qualitatively affect the ***dynamics*** of caste wage gaps between 1983 and 2012. We also find that the structural transformation that unfolded in the country during this period was important for the caste convergence. The shocks that changed the relative economic shares of the different sectors also changed the sectoral allocation of workers by caste thereby reducing the talent misallocation. Absent this sectoral churn, the misallocations would not have changed similarly.

markets in urban India. On a related theme, Ito (2009) studied labor market discrimination in two Indian states – Bihar and Uttar Pradesh. Exploring the theme of castes as networks, Munshi and Rosenzweig (2006) and Munshi and Rosenzweig (2016) show how caste networks impact labor mobility, education choices and employment. Our focus on aggregate caste dynamics and economic growth distinguishes our work from this literature.[7]

A second literature that is related to our work is the extensive work on structural transformation of countries along the development path wherein countries gradually switch their economic focus from agriculture to non-agricultural sectors. This is a voluminous literature that spans both empirical and theoretical work. Key contributions in this are Kongsamut et al. (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). An excellent overview of this literature can be found in Herrendorf et al. (2014). We differ from this work in our focus on the distributional effects of the transformation.

Our work also relates to the literature that has examined the effect of labor misallocations on productivity and growth. While this literature is long, the paper closest to our interest is the work on misallocation of talent by Hsieh et al. (2019) who analyze the consequences of misallocating talent by gender and race on productivity and growth in the USA. We share their interest in the implications of misallocating labor due to discrimination or other factors.

The next section describes the key facts on caste gaps in India. Section 3 presents the model and some analytical results. Section 4 presents the calibration and quantitative results; Section 5 uncovers the main mechanism at play while 6 provides and some independent evidence in support of the mechanism. In Section 7 we compute thee contribution of declining misallocation to productivity growth while Section 8 discusses our welfare results. Section 9 discusses issues related to affirmative action and structural transformation while the last section concludes.

2 Empirical regularities

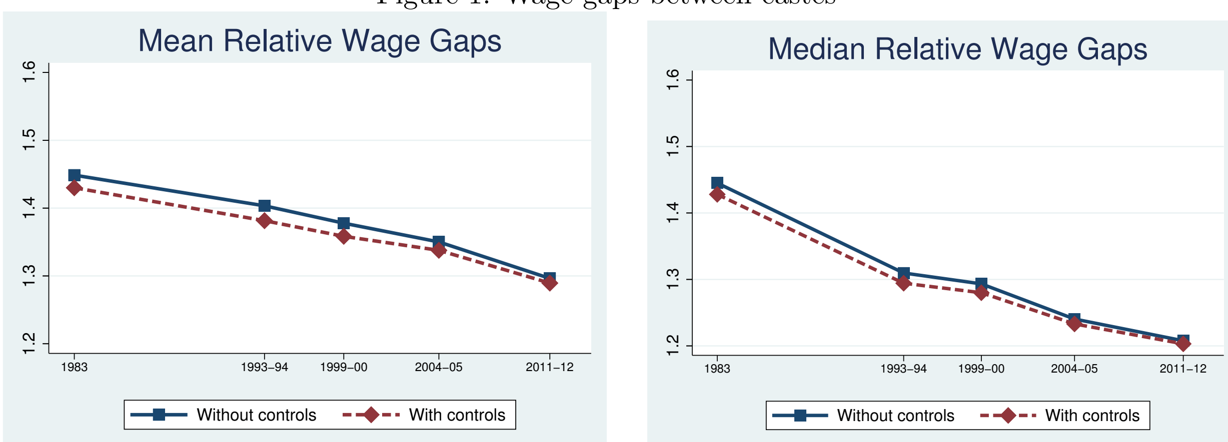
We start by reporting some aggregate and sectoral facts regarding the wage gaps between SC/STs and non-SC/STs during the period 1983-2012. These facts extend the results reported in Hnatkovska et al. (2012) to 2012 and provide additional cuts of the data. They serve as the empirical motivation

[7]Another paper that is related to our work is Banerjee and Munshi (2004). They examined the differences between entrants belonging to the incumbent traditional community of *Gounders* in the garment industry in Tirupur in India in the early 1990s relative to entrants from other communities. They found evidence of sharp catch-up of capital and output of outsider firms to the levels of entrants from the Gounder community.

of the paper. Our data mainly comes from various rounds of the National Sample Survey (NSS) employment-unemployment household surveys. Details on the data are contained in the Appendix.

Figure 1 reports the wage gaps between the castes across the NSS rounds. Panel (a) shows the mean wage gaps between the groups, while panel (b) shows the corresponding median gaps. The solid lines depict the unconditional wage gaps while the dashed lines show the wage gaps after controlling for the age characteristics of workers.[8] Both plots reveal an unambiguous pattern of wage convergence between the two groups since 1983, with the mean wage gap declining by 10.5% and the median gap falling by 14%.

Figure 1: Wage gaps between castes



(a) (b)

Notes: Panel (a) of this Figure presents the mean wage gaps between SC/STs and non-SC/STs (expressed as a ratio of non-SC/ST to SC/ST) from the 1983 to the 2011-12 NSS rounds. Panel (b) shows the corresponding median wage gaps. The dashed lines in the two panels show the computed wage gaps after controlling for the age characteristics of workers (age, age squared) while the solid lines are the gaps without such controls.

Given the trends in Figures 1, the natural question to ask is how much of the wage convergence can be explained by measurable worker characteristics like education as well as demographic variables such as age, gender, and location of residence? Hnatkovska et al. (2012) examined this question and found that most of the wage convergence is due to education convergence. We confirm their result during 1983-2012 by using DFL decompositions pioneered by DiNardo et al. (1996).

DFL decompositions consist of constructing a counterfactual wage density for non-SC/STs by re-weighting the non-SC/ST wage density with the distribution of SC/ST attributes of interest. We then compute a percentile wage gap between the actual and counterfactual wage densities for

[8]Specifically, to obtain unconditional wage gaps we estimated an OLS regression (for mean) and a Recentered Influence Function (RIF) regression (for median) of log wages on a constant and an SC/ST dummy. The conditional gaps are computed from the same regression with age and age squared controls.

non-SC/STs. This counterfactual percentile wage gap is then compared with the actual percentile gaps from the data to assess the role of the included attributes across the entire wage distribution. The closer the counterfactual wage gap to the actual gaps in the data, the greater is the explanatory power of the included attributes.

Figure 2: DFL Wage Decomposition

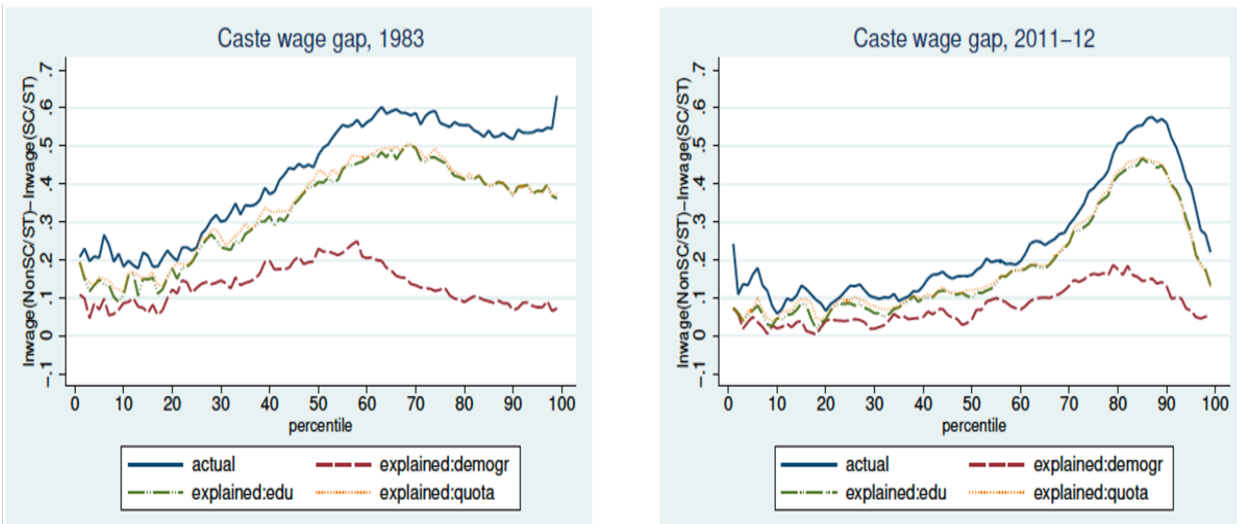


Figure 2 shows the DFL decompositions of the wage densities separately for 1983 and 2011-12 as well as the actual percentile wage gaps. For both years, we sequentially introduce three SC/ST attributes – demographic variables (age, age squared, gender, rural sector of residence, and region of residence), education, and state-level affirmative action benefits – to re-weight the non-SC/ST wage density.[9] Note that introduction of a new attribute is in addition to the previously included attributes. Thus, in Figure 2, comparing the schedule labelled "explained:edu" with the one labelled "explained:demogr" gives the marginal effect of education relative to just demographics in explaining the caste wage gap.

There are two main takeaways from Figure 2. First, in both 1983 and 2011-12, education explains over 90 percent of the wage difference between non-SC/STs and SC/STs across the entire distribution. In fact, neither demographics nor affirmative action account for much of the caste wage gap. Second, since the actual percentile wage gap shifted down in 2011-12 relative to 1983 in conjunction with the counterfactual percentile gap given by education, it follows that most of the

[9]We control for regional differences by grouping states into six regions-North, South, East, West, Central, and North-East to reflect similarities across states in their geographic characteristics. Education is introduced through a set of education dummies reflecting education categories 1-5. Affirmative action benefits are proxied by caste reservation quotas.

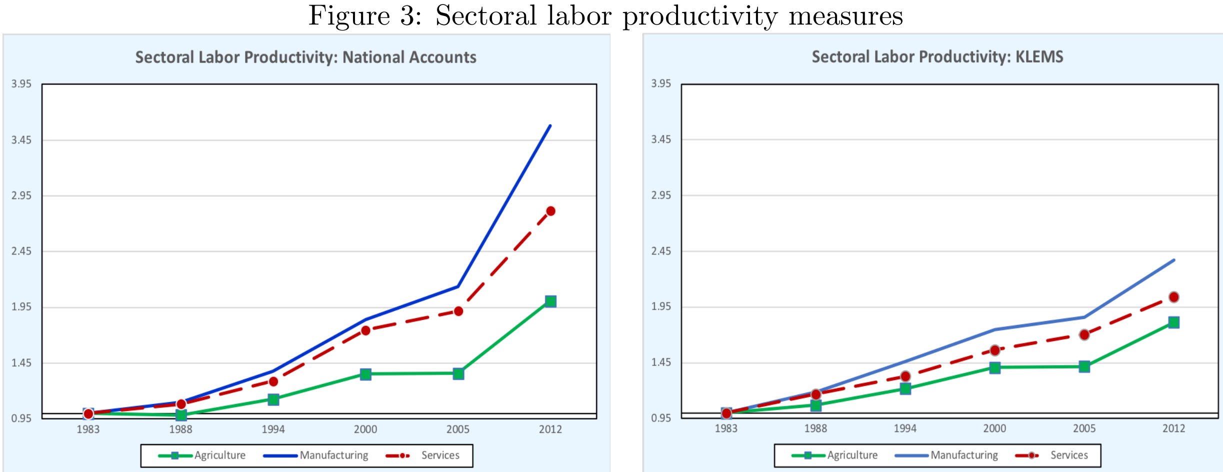
caste wage convergence in the data is accounted for by education convergence between the groups.

These trends raise the logical question about the deeper reasons behind the observed convergence between the groups during this period since education is clearly an endogenous choice. While there may have been multiple factors operating simultaneously, in this paper we focus on the two biggest changes that occurred in the Indian economy during this period. First, 1983-2012 saw a growth takeoff with average annual per capita GDP growth rising from 3.5 to 6 percent.

Second, this period was also marked by a structural transformation of the economy, with agricultural sector contracting and service sector expanding. There was an expansion in the manufacturing sector as well, but much more tepid relative to that of the service sector. See Appendix 11.1 for data and figures.

Much of this transformation was associated with rapid productivity growth across sectors as can be seen from Figure 3. Panel (a) shows sectoral output per worker computed from the national accounts data, while panel (b) shows the sectoral labor productivity as reported in the KLEMS dataset for India. All series are normalized by their values in 1983.

Figure 3: Sectoral labor productivity measures



(a) (b)

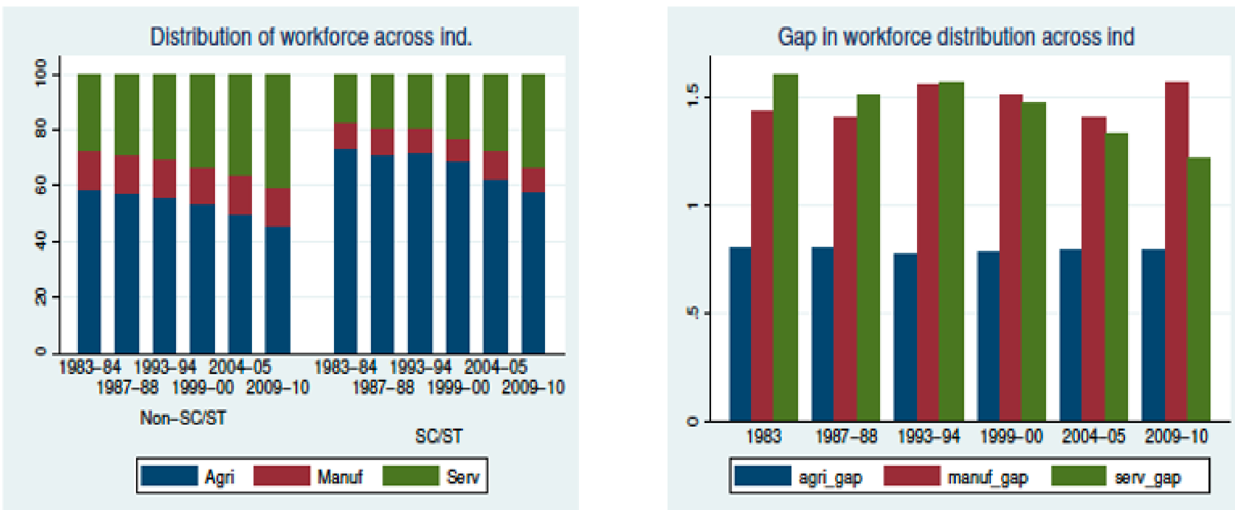
Notes: Panel (a) of this Figure presents labor productivity, measured as GDP (in constant 1980-81 prices) divided by number of workers in each sector. Panel (b) shows the sectoral labor productivity computed from the KLEMS database for India. All series are normalized by their 1983 values.

Both plots of Figure 3 also reveal a common rank-ordering of sectoral labor productivity growth during 1983-2012: manufacturing grew the fastest while agriculture was the slowest growing sector.

So, how did the structural transformation of India's economy affect the two social groups? Figure 4 reports the industry distribution of SC/ST and non-SC/ST workers, and the relative gaps in this distribution. SC/STs were and remain more likely to be employed in agriculture than non-

SC/STs. However the gap narrowed somewhat in the last ten years of our sample. The second largest industry of employment for both social groups is services, whose share has risen steadily over time. Interestingly, services also exhibits the sharpest convergence pattern between non-SC/STs and SC/STs with the relative gap in services employment shares shrinking from 60 percent in 1983 to 21 percent in 2012. Manufacturing shows relatively little change in the employment shares of the two groups over time.

Figure 4: Industry employment distribution across castes

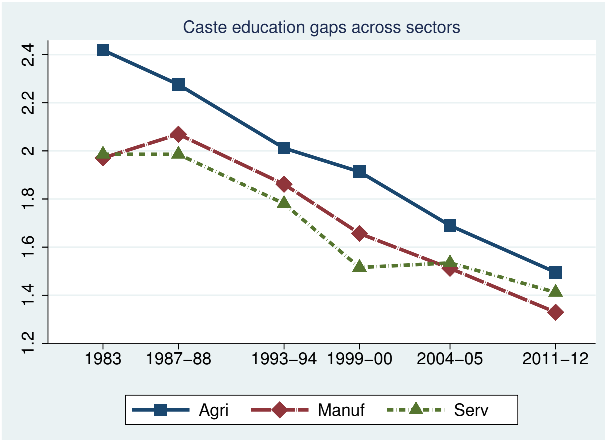


Notes: Panel (a) presents the distribution of workforce across the three industry categories for different NSS rounds. The left set of bars refers to non-SC/STs, while the right set is for SC/STs. Panel (b) presents the ratio of non-SC/STs to SC/STs shares reported in Panel (a) for each industry and year.

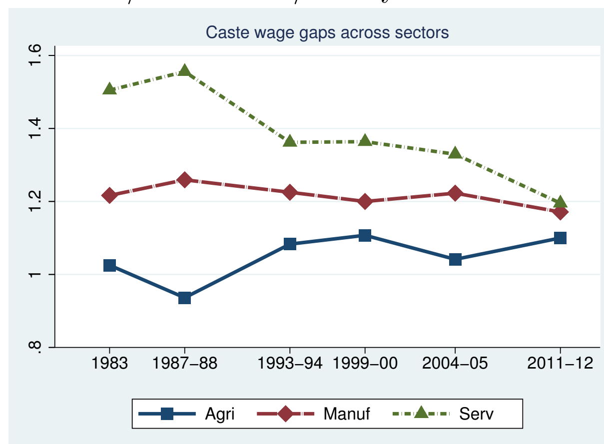
Figures 5 reports the relative gaps in education attainments and median wages between non-SC/STs and SC/STs employed in each sector. The education gaps have narrowed significantly over time between the two caste groups across all sectors. Median wage gaps, on the other hand declined in Services, stayed unchanged in Manufacturing, but widened somewhat in Agriculture.

To summarize, the period 1983-2012 was characterized by high aggregate growth and rising output per worker in all three sectors. There was a gradual transformation of the economy with the services share of both output and employment rising and the agriculture share shrinking. The education gap between the castes declined in all three sectors. While wages were converging *overall* between the castes, there were contrasts in the sectoral patterns: the wage convergence was strong in the service sector but the agricultural sector saw a wage divergence.

Figure 5: Education and wage gaps between non-SC/STs and SC/STs by sector



(a)



(b)

Notes: Panel (a) presents relative gap in years of education between non-SC/STs and SC/STs in three sectors. Panel (b) presents the ratio of non-SC/STs median wages to SC/STs median wages in three sectors.

3 Model

We now ask whether productivity shocks can have a differential impact on the two groups and cause the education and wage gaps between the castes to fall? If so, what are the conditions under which that can happen? Would such an environment also induce sectoral outcomes that are consistent with the facts that we outlined above?

Consider a one-period lived closed economy that is inhabited by a continuum of agents of measure L. A measure S of these agents belong to caste s (for scheduled castes and tribes or SC/STs) while a measure $N = L - S$ belong to caste n for non-SC/ST.

Individuals belonging to different castes will be distinct along two margins: the cost of acquiring schooling and the cost of accessing sectoral labor markets. We shall elaborate on each of these features below.

An agent i belonging to caste $j = n, s$ maximizes utility from consumption of the final good

$$u\left(c_{ij}\right) = \frac{c_{ij}^{1-\rho}}{1-\rho}$$

Agents produce a final good by combining three intermediate goods using the technology

$$y_{ij} = \left(y_{ij}^{a} - \bar{y}\right)^{\theta} \left(y_{ij}^{m}\right)^{\eta} \left(y_{ij}^{h}\right)^{1-\theta-\eta}$$

where y^k is intermediate good $k = a, m, h$. In the following we shall refer to the a good as the agricultural good, the m good as the manufacturing good and the h good as the high skill good. $\bar{y}$ is the minimum required level of the a good.[10]

Intermediate goods are acquired by agent i using her income w_i. Specifically, an agent i of caste $j = n, s$ with income w_{ij} chooses y^a, y^m, y^h to maximize production of the final good y subject to the budget constraint

$$p^a y_{ij}^a + p^m y_{ij}^m + p^h y_{ij}^h = w_{ij}$$

The optimal expenditures on intermediate goods by an agent i are

$$p^a y_{ij}^a = \theta \left(w_{ij} - p^a \bar{y} \right) + p^a \bar{y} \tag{3.1}$$

$$p^m y_{ij}^m = \eta \left(w_{ij} - p^a \bar{y} \right) \tag{3.2}$$

$$p^a y_{ij}^a = \left(1 - \theta - \eta \right) \left(w_{ij} - p^a \bar{y} \right) \tag{3.3}$$

Substituting the optimal intermediate goods purchases into the production function for the final good gives

$$y_{ij} = \frac{\theta^\theta \eta^\eta (1 - \theta - \eta)^{1-\theta-\eta}}{p^{a\theta} p^{m\eta} p^{h(1-\theta-\eta)}} \left(w_{ij} - p^a \bar{y} \right)$$

We define the aggregate price index in this economy (the unit cost of producing the final good) as

$$P = \frac{(p^a)^\theta (p^m)^\eta (p^h)^{1-\theta-\eta}}{\theta^\theta \eta^\eta (1 - \theta - \eta)^{1-\theta-\eta}} \tag{3.4}$$

Since we use the final good as the numeraire, with no loss of generality, we set $P = 1$ throughout the model. Hence, the optimal production of the final good by agent i belonging to caste $j = n, s$ is

$$y_{ij} = w_{ij} - p^a \bar{y} \tag{3.5}$$

The non-homotheticity in production of the final good due to a minimum use of the agricultural good will be one source of structural transformation in the model.

[10] The general homothetic CES specification induces structural transformation towards the slowest growing sector under the assumption of the elasticity of substitution between sectors being less than one. This has been used by a number of authors to generate the observed structural transformation in the industrial countries (see Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008)). But that specification would induce a counterfactual expansion of the agricultural sector in India since it was the slowest growing sector during 1983-2012. We proceed with a non-homothetic Cobb-Douglas aggregator instead as it generates the observed structural transformation in India without us having to take a sharp stand on the specific value of the inter-sectoral elasticity of substitution.

3.1 Ability and Human Capital

Each agent is born with an endowment of ability a_i and one unit of labor time that is supplied inelastically to the labor market. Ability is drawn from an *i.i.d.* process that follows the cumulative distribution function $G(a)$, $a \in [\underline{a}, \bar{a}]$. The ability distribution is identical for both castes.

Ability is a productive input in building human capital. Human capital, in turn, determines the agent's labor productivity as well as the cost of accessing sector specific labor markets. Specifically, human capital of an agent i is determined by

$$e_{ij} = a_{ij} q_{ij}^{\chi}, \ \chi \in (0,1) \tag{3.6}$$

where q is schooling acquired by the agent and χ denotes the schooling elasticity of human capital.

Acquiring human capital is expensive with the cost of acquiring q units of schooling being $\lambda_j q$ where λ_j, $j = n, s$ is the marginal cost of schooling which is denominated in terms of the final good. Note that the marginal cost of education is constant and caste specific. This is the first difference between individuals belonging to different castes.

3.2 Human Capital and Sectoral Employment

An agent can work in any of the three sectors conditional on paying the entry costs of accessing those sectors. With no loss of generality, we normalize the entry cost in sector-a to zero. Access to sectors m and h however are costly. Agent i can access sector-$k = m, h$ by spending f_{ij}^k units of the final good. Notice that this specification allows the sectoral entry costs to be caste specific.

In what follows we shall make the following assumptions:

Assumption 1:

$$f_{ij}^k = \begin{cases} 0, & k = a; \ j = n, s \\ \phi(\gamma_j^k - \alpha e_{ij}), & k = m, h; \ j = n, s \end{cases}$$

Assumption 2: $\gamma_j^h > \gamma_j^m, \quad j = n, s$

Assumption 1 says that sectoral entry costs only apply for entry into sectors m and h. The entry costs have two components. The first, γ_j^k, is a fixed cost that is specific to sector and caste. The second component, αe_{ij}, is decreasing in the human capital of the individual but where the marginal effect of human capital on the entry cost is identical across castes. ϕ is a scaling factor that has no qualitative effect on the results but is useful for quantitative purposes.[11]

Assumption 2 implies that the fixed cost of entry into sector-h is greater than the entry cost for sector-m for both castes. This ensures an ability rank order where the highest ability individuals work in sector-h (consistent with the evidence on the sectoral distribution of human capital).

The preceding makes clear that there are two fundamental sources of differences across castes: the cost of education λ and the fixed costs of entry into sectors m and h. We shall explore the implications of these differences below.

3.3 Sectoral Production Technologies

The technologies for producing the three goods are all linear in the human capital of the worker. In particular, a worker with ability e_i supplying one unit of labor time to sector a produces

$$y_i^a = Ae_i$$

An m-sector worker with ability e_i produces the manufacturing good m according to

$$y_i^m = Me_i$$

Lastly, an $h-$sector worker with ability e_i produces the high skill good according to

$$y_i^h = He_i$$

Note that labor supply is inelastic and indivisible. So each worker supplies one unit of labor time to whichever sector she works in.

3.4 Sector and Schooling Choice

The decisions about which sector to work in and what human capital level to acquire are joint in this model since the schooling decision is contingent on the returns to human capital which, in turn, is dependent on the sector of employment of the worker since human capital impacts both

[11] The second component of the sectoral entry cost, αe_{ij}, is not required for any of the qualitative results on caste gaps that we derive below. However, we allow for this second term, which is independent of caste, to allow for the fact that schooling creates network of connections that is broader than the individual's immediate family and caste connections or networks.

the direct returns to work as well as the sectoral entry costs.

3.4.1 The schooling choice

An agent belonging to caste $j = n, s$ who intends to work in sector-a will choose schooling q to maximize to maximize consumption:

$$c_{ij}^{a} = y_{ij}^{a} - \lambda_j q_{ij}$$

Similarly, an agent planning to work in sector-m will choose her schooling to maximize

$$c_{ij}^{m} = y_{ij}^{m} - \lambda_j q_{ij} - \phi\left(\gamma_j^{m} - \alpha a_{ij} q_{ij}^{\chi}\right)$$

while an agent headed for work in sector-h would choose schooling q to maximize

$$c_{ij}^{h} = y_{ij}^{h} - \lambda_j q_{ij} - \phi\left(\gamma_j^{h} - \alpha a_{ij} q_{ij}^{\chi}\right)$$

where $y_{ij}^{k} = w_{ij}^{k} - p^{a}\bar{y}, k = a, m, h$. w_{ij}^{k} denotes wages for the individual contingent on the sector that she chooses to work in. These sectoral wages are given by

$$w_{ij}^{k} = \begin{cases} p^{a} A a_{ij}\left(q_{ij}^{a}\right)^{\chi} & \text{if } i \text{ works in } a \\ p^{m} M a_{ij}\left(q_{ij}^{m}\right)^{\chi} & \text{if } i \text{ works in } m \\ p^{h} H a_{ij}\left(q_{ij}^{h}\right)^{\chi} & \text{if } i \text{ works in } h \end{cases}$$

Notice that the schooling choice contingent on working in sector $k = a, m, h$ internalizes the effects of schooling on the sectoral entry costs.

The optimal schooling choices for an agent i belonging to caste j who chooses to work in sector-$k = a, m, h$ are:

$$q_{ij}^{a} = \left(\frac{\chi a_{ij} p^{a} A}{\lambda_j}\right)^{\frac{1}{1-\chi}} \tag{3.7}$$

$$q_{ij}^{m} = \left(\frac{\chi a_{ij}\left(p^{m} M + \phi\alpha\right)}{\lambda_j}\right)^{\frac{1}{1-\chi}} \tag{3.8}$$

$$q_{ij}^{h} = \left(\frac{\chi a_{ij}\left(p^{h} H + \phi\alpha\right)}{\lambda_j}\right)^{\frac{1}{1-\chi}} \tag{3.9}$$

The optimal schooling functions above reflect two key features. First, within each sector higher ability agents acquire more schooling and hence, greater human capital. Second, controlling for ability, sectors with higher labor productivity will have workers with greater human capital since schooling is increasing in sectoral productivity.

3.4.2 Sectoral employment choice

The decision regarding the sector of employment is then based on choosing the sector associated with the highest consumption: $max\left\{c_{ij}^a, c_{ij}^m, c_{ij}^h\right\}$ where c_{ij}^k denotes the consumption of an agent i of caste j working in sector $k = a, m, h$. Since both schooling and sectoral entry costs are paid out of the household final good, the household budget constraint dictates that $c_{ij}^k = y_{ij}^k - \lambda_j q_{ij}^k - f_{ij}^k$ where y_{ij}^k is given by equation 3.5 and f_{ij}^k is given by Assumption 1.

The sector-specific schooling levels in equations 3.7-3.9 above imply consumption levels for agents contingent on their decisions regarding schooling and sector of employment:

$$c_{ij}^a = (1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \left(a_{ij} p^a A\right)^{\frac{1}{1-\chi}} - p^a \bar{y} \tag{3.10}$$

$$c_{ij}^m = (1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \left\{a_{ij}\left(p^m M + \phi\alpha\right)\right\}^{\frac{1}{1-\chi}} - \phi\gamma_j^m - p^a \bar{y} \tag{3.11}$$

$$c_{ij}^h = (1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \left\{a_{ij}\left(p^h H + \phi\alpha\right)\right\}^{\frac{1}{1-\chi}} - \phi\gamma_j^h - p^a \bar{y} \tag{3.12}$$

As in the schooling decisions, consumption of agents is also increasing in their ability a within each sector. Note that the consumption levels associated with working in each sector are net of the costs of schooling and sectoral entry costs since those are paid by the agent out of the household final good y_{ij}.

To describe the distribution of agents into the different sectors it is useful to define three ability thresholds:

$$\hat{a}_j^m = \left[\frac{\phi\gamma_j^m}{(1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}}\left\{\left(p^m M + \phi\alpha\right)^{\frac{1}{1-\chi}} - \left(p^a A\right)^{\frac{1}{1-\chi}}\right\}}\right]^{1-\chi} \tag{3.13}$$

$$\hat{a}_j^h = \left[\frac{\phi\gamma_j^h}{(1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}}\left\{\left(p^h H + \phi\alpha\right)^{\frac{1}{1-\chi}} - \left(p^a A\right)^{\frac{1}{1-\chi}}\right\}}\right]^{1-\chi} \tag{3.14}$$

$$\tilde{a}_j^h = \left[\frac{\phi(\gamma_j^h - \gamma_j^m)}{(1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \left\{ (p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - (p^m M + \phi\alpha)^{\frac{1}{1-\chi}} \right\}} \right]^{1-\chi} \tag{3.15}$$

Equation 3.13 defines the threshold ability level $\hat{a}^m$ for which consumption from working in sector-a is the same as consumption from working in sector-m, i.e., $c_{ij}^a = c_{ij}^m$. Hence, an agent with ability $\hat{a}^m$ is indifferent between working in sector-a or sector-m. $\hat{a}^h$ and $\tilde{a}^h$ give the corresponding indifference between sectors-a and h, and between sectors m and h, respectively.

We now make the following assumption to provide greater structure to the cross-sectoral distribution of ability and skills that the model can generate:

Assumption 3: Parameter values guarantee $p^h H + \phi\alpha > p^m M + \phi\alpha > p^a A$

Assumption 3 is necessary (but not sufficient) for there to be a distribution of abilities across all three sectors. This will become clearer in the analysis below.

The thresholds along with Assumptions 1-3 allow a clear pairwise ranking of sectors for each ability type. This is summarized in the following Lemma:

Lemma 3.1. *All individuals $i \in$ caste $j = n, s$ with ability a_{ij} prefer employment in sector-m to employment in sector-a if $a_{ij} \geq \hat{a}_j^m$; employment in sector-h to sector-a if $a_{ij} \geq \hat{a}_j^h$; and employment in sector-h to sector-m if $a_{ij} \geq \tilde{a}_j^h$.*

Proof. See Appendix. ■

3.4.3 Mapping Abilities to Sectors

How do agents get distributed across sectors in this economy? This depends on the relative rank ordering of the three thresholds $\hat{a}_j^m, \hat{a}_j^h$, and $\tilde{a}^h$. The following lemma is useful for characterizing the different possibilities:

Lemma 3.2. *The rank order of the three ability thresholds are*

$$\tilde{a}_j^h < \hat{a}_j^h < \hat{a}_j^m \quad if \quad \hat{a}_j^h = min[\hat{a}_j^m, \hat{a}_j^h]$$
$$\tilde{a}_j^h > \hat{a}_j^h > \hat{a}_j^m \quad if \quad \hat{a}_j^h = max[\hat{a}_j^m, \hat{a}_j^h]$$

Proof. See Appendix. ■

Lemma 3.2 describes the relationship between the three thresholds in the model. Specifically, it says that $\tilde{a}_j^h$ cannot lie in between $\hat{a}_j^m$ and $\hat{a}_j^h$. Rather, it lies on the same side of $\hat{a}_j^m$ as $\hat{a}_j^h$.

Since the model structure can give rise to $\hat{e}^h \gtrless \hat{e}^m$, the following Proposition characterizes the mapping of the abilities to sectoral employment under both these cases:

Proposition 3.1. *(a) When $\hat{a}_j^h > \hat{a}_j^m$, $j = n, s$, the sectoral distribution of abilities is*

$$a_i \in \begin{cases} [\underline{a}_j, \hat{a}_j^m) & : \ i \in A \\ [\hat{a}_j^m, \tilde{a}_j^h) & : \ i \in M \\ [\tilde{a}_j^h, \bar{a}_j] & : \ i \in H \end{cases}$$

b) When $\hat{a}_j^h < \hat{a}_j^m$, $j = n, s$, the sectoral distribution of abilities is

$$a_i \in \begin{cases} [\underline{a}_j, \hat{a}_j^h) & : \ i \in A \\ [\hat{a}_j^h, \hat{a}_j^m) & : \ i \in H \\ [\hat{a}_j^m, \bar{a}_j] & : \ i \in H \end{cases}$$

Proof. (a) When $\hat{a}_j^m < \hat{a}_j^h$, Lemma 3.2 says that we must have $\hat{a}_j^m < \hat{a}_j^h < \tilde{a}_j^h$. The distribution of ability types across the three sectors in this case follows directly from equations 3.13, 3.14, 3.15, and Lemma 3.1. Ability types below $\hat{a}_j^m$ work in sector-a while those in between $\hat{a}_j^m$ and $\hat{a}_j^h$ choose sector-m. For ability types between $\hat{a}_j^h$ and $\tilde{a}_j^h$, equation 3.15 implies that employment in sector-m is strictly preferred to sector-h. Those with ability above $\tilde{a}_j^h$ choose to work in sector-h, which follows directly from equation 3.15.

(b) When $\hat{a}_j^h < \hat{a}_j^m$, from Lemma 3.2 we have $\tilde{a}_j^h < \hat{a}_j^h < \hat{a}_j^m$. In this case, the distribution of ability types across sectors follows directly from equations 3.13-3.14 and Lemma 3.1. Ability types below $\hat{a}_j^h$ strictly prefer employment in sector-a to both sectors h and m. For all ability types in caste $j = n, s$ with $a \in [\hat{a}_j^h, \hat{a}_j^m)$, employment in sector-$h$ dominates both sectors a and m. For $a \geq \hat{a}_j^m > \tilde{a}_j^h$, equation 3.13 says that sector-m dominates sector-a while equation 3.15 says that working in sector-h is strictly preferred by these types over sector-m employment. ■

While the message of Proposition 3.1 is self-explanatory, a comment on part (b), which describes allocations when $\hat{a}_j^h < \hat{a}_j^m$, is useful. The ability distribution described in Proposition 3.1 for this case implies that labor from both castes choose employment in either sector-a or sector-h, thereby rendering sector-m empty. This is clearly counterfactual since our data analysis revealed that both castes were employed in all three sectors. In the remainder of the paper we ignore this case and focus exclusively on parameter configurations such that $\hat{a}_j^h > \hat{a}_j^m$ for $j = n, s$.[12]

3.5 Market clearing and Equilibrium

Markets for each good must clear individually. For the intermediate goods, this implies that total production must equal total demand for each good individually:

$$Y^a = L\left[s\int_{\underline{a}}^{\bar{a}} y_{is}^a dG(a) + n\int_{\underline{a}}^{\bar{a}} y_{in}^a dG(a)\right] \tag{3.16}$$

$$Y^m = L\left[s\int_{\underline{a}}^{\bar{a}} y_{is}^m dG(a) + n\int_{\underline{a}}^{\bar{a}} y_{in}^m dG(a)\right] \tag{3.17}$$

$$Y^h = L\left[s\int_{\underline{a}}^{\bar{a}} y_{is}^h dG(a) + n\int_{\underline{a}}^{\bar{a}} y_{in}^h dG(a)\right] \tag{3.18}$$

where Y^k denotes total production of intermediate good $k = a, m, h$. Note that in equations 3.16-3.18, sectoral output of individual i belonging to caste $j = n, s$ whose ability is outside the relevant sectoral ability thresholds given in Proposition 3.1 will be zero.

Total production of the final good must equal the total demand for the final good:

$$C + Q + F = Y = L\left[s\int_{\underline{a}}^{\bar{a}} y_{is} dG(a) + n\int_{\underline{a}}^{\bar{a}} y_{in} dG(a)\right] \tag{3.19}$$

where Q denotes total costs of schooling by all workers, F denotes the total skill acquisition costs incurred by workers employed in sector m and sector h, while Y denotes total production of the final good by all agents.. The market clearing condition for the m good recognizes that part of the use of the good is for acquiring skills.

DEFINITION: *The Walrasian equilibrium for this economy is a vector of prices* $\{p_m, p_h\}$ *and quantities* $\{Y^a, Y^m, Y^h, C_s, C_n, Q_s, Q_n, F^m, F^h,\ \hat{a}_s^m, \hat{a}_s^h, \hat{a}_n^m, \hat{a}_n^h\}$ *such that all worker-households satisfy their optimality conditions, budget constraints are satisfied and all markets clear.*

3.6 Sectoral Labor and Wage Gaps Between Castes

It is useful at this stage to describe the caste labor gaps and wage gaps in the three sectors since those are a key object of interest. The precise expressions for these gaps depend on the specifics of

[12]The case $\hat{a}_j^h = \hat{a}_j^m = \tilde{a}_j^j$ is possible but clearly non-generic. Consequently, we ignore this pathological possibility.

the underlying distribution from which individuals draw their ability endowment. Throughout the rest of the paper we shall maintain the assumption that the ability distribution is uniform:

Assumption 4: The ability distribution $G(a)$ is uniform on the support $[\underline{a}, \bar{a}]$.

The labor employment gap between caste n and caste s in sector $k = a, m, h$ is the ratio of the fraction of caste n workers employed in sector k to the fraction of caste s workers employed in sector k. Under Assumption 4, these gaps are given by:

$$\Delta s^a = \frac{\hat{a}_n^m - \underline{a}}{\hat{a}_s^m - \underline{a}} \tag{3.20}$$

$$\Delta s^m = \frac{\tilde{a}_n^h - \hat{a}_n^m}{\tilde{a}_s^h - \hat{a}_s^m} \tag{3.21}$$

$$\Delta s^h = \frac{\bar{a} - \tilde{a}_n^h}{\bar{a} - \tilde{a}_s^h} \tag{3.22}$$

To derive the sectoral caste wage gaps from the model, note that the ability thresholds and the sector-contingent schooling choices given by equations 3.7-3.9 imply that the mean sectoral wages of agents belonging to caste $j = n, s$ are

$$w_j^a = (p^a A)^{\frac{1}{1-\chi}} \left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \int_{\underline{a}}^{\hat{a}_j^m} a^{\frac{1}{1-\chi}} \frac{dG(a)}{G(\hat{a}_j^m)}$$

$$w_j^m = p^m M \left(p^m M + \phi\alpha\right)^{\frac{\chi}{1-\chi}} \left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \int_{\hat{a}_j^m}^{\tilde{a}_j^h} a^{\frac{1}{1-\chi}} \frac{dG(a)}{G(\tilde{a}_j^h) - G(\hat{a}_j^m)}$$

$$w_j^h = p^h H \left(p^h H + \phi\alpha\right)^{\frac{\chi}{1-\chi}} \left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \int_{\tilde{a}_j^h}^{\bar{a}} a^{\frac{1}{1-\chi}} \frac{dG(a)}{1 - G(\tilde{a}_j^h)}$$

Since the caste wage gap in sector $k = a, m, h$ is the ratio of the mean wage of caste n relative to the mean wage of caste s in sector k, the sectoral caste wage gaps under Assumption 4 are given by:

$$\Delta w^a = \left(\frac{\lambda_s}{\lambda_n}\right)^{\frac{\chi}{1-\chi}} \left(\frac{(\hat{a}_n^m)^{\frac{1}{1-\chi}+1} - (\underline{a})^{\frac{1}{1-\chi}+1}}{(\hat{a}_s^m)^{\frac{1}{1-\chi}+1} - (\underline{a})^{\frac{1}{1-\chi}+1}}\right) \left(\frac{\hat{a}_s^m - \underline{a}}{\hat{a}_n^m - \underline{a}}\right) \tag{3.23}$$

$$\Delta w^m = \left(\frac{\lambda_s}{\lambda_n}\right)^{\frac{\chi}{1-\chi}} \left(\frac{(\tilde{a}_n^h)^{\frac{1}{1-\chi}+1} - (\hat{a}_n^m)^{\frac{1}{1-\chi}+1}}{(\tilde{a}_s^h)^{\frac{1}{1-\chi}+1} - (\hat{a}_s^m)^{\frac{1}{1-\chi}+1}}\right) \left(\frac{\tilde{a}_s^h - \hat{a}_s^m}{\tilde{a}_n^h - \hat{a}_n^m}\right) \tag{3.24}$$

$$\Delta w^h = \left(\frac{\lambda_s}{\lambda_n}\right)^{\frac{\chi}{1-\chi}} \left(\frac{\bar{a}^{\frac{1}{1-\chi}+1} - (\tilde{a}_n^h)^{\frac{1}{1-\chi}+1}}{\bar{a}^{\frac{1}{1-\chi}+1} - (\tilde{a}_s^h)^{\frac{1}{1-\chi}+1}}\right) \left(\frac{\bar{a} - \tilde{a}_s^h}{\bar{a} - \tilde{a}_n^h}\right) \tag{3.25}$$

where the thresholds $\hat{a}_j^m, \tilde{a}_j^h$ are given by equations 3.13 and 3.15, respectively.

The wage and labor expressions above make clear that the key variables that determine the sectoral caste gaps in the model are the ability thresholds $\hat{a}_j^m$ and $\tilde{a}_j^h$ for $j = n, s$. The differences in the ability thresholds across the castes, in turn, depend on differences in schooling costs and sectoral entry costs. This follows directly from equations 3.13 and 3.15 which can be used to get

$$\frac{\hat{a}_n^m}{\hat{a}_s^m} = \left(\frac{\lambda_n}{\lambda_s}\right)^{\chi} \left(\frac{\gamma_n^m}{\gamma_s^m}\right)^{1-\chi} \quad (3.26)$$

$$\frac{\tilde{a}_n^h}{\tilde{a}_s^h} = \left(\frac{\lambda_n}{\lambda_s}\right)^{\chi} \left(\frac{\gamma_n^h - \gamma_n^m}{\gamma_s^h - \gamma_s^m}\right)^{1-\chi} \quad (3.27)$$

These results show that the ability thresholds as well as the education and employment distributions differ across the castes in the model despite members of the two castes drawing from the same ability distribution. These caste gaps arise due to differences in the costs of schooling and the sectoral entry fixed costs which are the only sources of difference across castes in the model.

4 A Quantitative Evaluation

We now turn to a quantitative implementation of the full version of the three-sector model. Specifically, we examine whether a calibrated version of the three sector model can explain the observed caste gap dynamics through the observed macroeconomic growth; and whether the caste education subsidization in India through reservations were crucial for the observed convergence.

The quantitative strategy of this section is to calibrate the model to the mimic the 1983 distribution of education, sectoral employment and sectoral wage of the two castes. Next, we identify the sectoral productivity changes between 1983 and 2012 by matching the change in sectoral labor productivities in the model with the corresponding changes in the sectoral output per unit labor reported in the National Income and Product Accounts data. We then feed the estimated paths of sectoral productivity into the calibrated model. The resulting distributional implications of the model at each date are then compared to the data in order to evaluate the explanatory power of aggregate productivity shocks for the caste wage gap dynamics.

Our focus is on eight key data moments for 1983: the three sectoral caste employment gaps; the three sectoral caste wage gaps; and the two average education levels $\bar{q}_n$ and $\bar{q}_s$. Our calibration strategy is to match these eight data moments by choosing the following eight parameters: the sectoral entry cost parameters $\left(\gamma_s^m, \gamma_s^h, \frac{\gamma_n^m}{\gamma_s^m}, \frac{\gamma_n^h}{\gamma_s^h}\right)$, the two education cost parameters $\left(\frac{\lambda_n}{\lambda_s}, \lambda_s\right)$, the

Table 1: Calibration of Key Variables

VARIABLE	VALUE	VARIABLE	VALUE
$\underline{c}$	0.5	θ	0.46
η	0.15	α	1
$\underline{a}$	1	$\bar{a}$	50
$\frac{M}{A}_{1983}$	1.2	$\frac{H}{A}_{1983}$	1.1
L	1	S	0.25
Calibrated variables			
γ_s^m	20.1360	γ_s^h	299.1381
$\frac{\gamma_n^m}{\gamma_s^m}$	1.036	$\frac{\gamma_n^h-\gamma_n^m}{\gamma_s^h-\gamma_s^m}$	1.332
$\frac{\lambda_s}{\lambda_n}$	1.55	ϕ	0.53
λ_s	2.53	χ	0.61

Notes: The table gives the parameters used for calibrating the model. The top panel lists the parameter values that were taken from other studies. The parameters in the bottom panel of the table were picked to match data moments from 1983.

scaling parameter ϕ and the schooling elasticity of human capital χ.

Table 1 reports the key parameters. The upper panel of the table gives the parameters that were either normalizations or values that were taken from other studies. The numbers in the lower panel are the ones that were calibrated to match the moments of the 1983 caste distribution.

There are two features to note about the calibration parameters in Table 1. First, in order to match the sectoral caste gaps in 1983 the model demands that $\frac{\lambda_s}{\lambda_n} = 1.55$ so that the schooling costs for caste-s are 55 percent higher than that for caste-n. This feature allows the model to match the fact that SC/STs are over-represented in sector-a. The higher cost of schooling limits their access to the non-agricultural sectors.[13]

Second, matching the caste gaps in 1983 also requires the fixed costs of entry into sectors m and h to be lower for the disadvantaged caste-s. Intuitively, given the schooling gap between SC/STs and non-SC/STs, the model would predict counterfactually few SC/STs in the higher skill

[13]The higher schooling costs for SC/STs reduces the share of SC/ST who transit to the higher skill sectors, thereby raising the average ability of SC/STs in agriculture. However the lower levels of schooling of the SC/STs who remain in Agriculture lowers the labor productivity of SC/ST workers in Agriculture enough to allow the model to simultaneously generate $\Delta s^A < 1$ and $\Delta w^A > 1$.

sectors. The higher schooling costs faced by SC/STs would also *reduce* the average wage gap in manufacturing and services since only the most high ability and highly educated SC/STS would work in those sectors. But that is counterfactual since the wage gaps in these two sectors are higher than in agriculture. Having lower sectoral entry costs for SC/STs in manufacturing and services allows the model to simultaneously match the sectoral employment and wage gaps between the castes in the data.

Lower sectoral entry costs for SC/STs are consistent with the presence of affirmative action programs that provide reservations for SC/STs in public sector jobs, which are mostly in the non-agricultural sectors. Indeed, in the rest of the analysis we interpret these lower entry costs as affirmative action policies.

Our quantification strategy is to freeze the calibrated parameters at the 1983 values and recompute the equilibrium by feeding in the identified change in the exogenous sectoral productivities A, M, H between 1983 and 2012. Note that since the model has no intrinsic dynamics, each new level for productivity generates a new equilibrium.

Given the specification of our model, one cannot compute the exogenous sectoral productivities from the sectoral labor productivities reported in the National Income accounts. In the model, agents endogenously acquire human capital through schooling and also choose their sector of employment. This educational and sectoral sorting impacts their productivity. Consequently, sectoral output per unit of sectoral labor would reflect the joint effects of exogenous sectoral productivity, endogenous human capital of the sectoral labor force and the endogenous sectoral sorting by workers. This is true both in the data and the model.

We approach the problem in a hybrid way. We first estimate sectoral productivities in 1983 by running sectoral Mincer wage regressions on five categories of education attainment of workers (primary, middle, secondary, college, diploma/technical) and a constant using the NSS employment/unemployment household survey for 1983. We use the constant in these sectoral wage regressions as our estimates of sectoral productivity in 1983. These numbers for relative sectoral productivities are reported in the top panel of Table 1.

To get the growth rates between 1983 and 2012 of the exogenous sectoral productivities A, M, and H, we first fix the calibrated parameters at 1983 level. We then pick the exogenous sectoral productivity growth rates such that the implied growth rates of sectoral output per worker between 1983 and 2012 in the model exactly match the corresponding growth rates in the data.[14] This

[14] In our data analysis, labor productivity is computed as average output per worker for each sector in 1983 prices

procedure yields the following sectoral productivity growth rates[15] :

$$g_A = 1.1436 \quad g_M = 2.1421 \quad g_H = 2.4068$$

Table 2 shows the match between the eight targeted variables and their corresponding data values in 1983. The model clearly matches the rank order and magnitudes of the targeted moments for the sectoral caste gaps in both labor shares and wages gaps. It also does well in matching the mean education levels of the two castes in 1983, though the fit is not quite as precise as that for the six sectoral caste gaps.[16]

Table 2: Data and Model Moments

		1983		2012	
Variable	Notation	Data	Model	Data	Model
		Targeted		Non-Targeted	
Wage Gap Agriculture	Δw^a	1.04	1.04	1.08	1.05
Wage Gap Manufacture	Δw^m	1.20	1.20	1.14	1.20
Wage Gap Service	Δw^h	1.45	1.45	1.33	1.16
Labor Share Gap Agri	Δs^a	0.80	0.85	0.79	0.85
Labor Share Gap Manuf	Δs^m	1.43	1.43	1.57	2.15
Labor Share Gap Serv	Δs^h	1.61	1.60	1.21	1.32
Mean educ SC/ST	$\bar{q}_s$	1.81	1.75	4.73	3.78
Mean educ Non-SC/ST	$\bar{q}_n$	4.08	3.86	5.78	6.59
		Non-Targeted		Non-Targeted	
Total wage gap	Δw	1.45	1.34	1.30	1.24
Pareto shape para: Schooling SC/ST	k_s	0.57	0.77	1.33	1.19
Pareto shape para: Schooling Non-SC/ST	k_n	1.12	1.16	1.52	1.58

Notes: The top panel of the table reports the sectoral caste gaps in employment and wages with all gaps being the ratio of Non-SC/ST to SC/ST. The bottom panel reports the data and model generated of selected non-targeted moments.

How well does the model perform with respect to the non-targeted moments for the two castes in 1983? The bottom panel of Table 2 shows the fit of the model with respect to three non-targeted caste gaps. The first is the one that is the main object of the paper: the overall caste wage gap. The model generates a relative wage premium for non-SC/STs of 34 percent. Relative to the 45

whose model counterpart is $Ey^k = \frac{p^k_{83} Y^k}{L^k} \quad k = a, m, h$ where Y^k is given by (3.16)-(3.18), p^k_{83} is price levels at 1983, and L^k is the measure for employment in sector k.

[15]The fact that the estimated agricultural labor productivity growth in India during 1983-2012 is the lowest amongst the three sectors is a pattern that is echoed also in the growth of overall sectoral output per worker during this period. We expand on these sectoral productivities and their implications for sectoral prices in Section 7 below.

[16]Schooling in the model is a continuous variable whereas in the data it is in years of schooling. To compare the schooling statistics in the model with the data, we normalize both the model and the data education series by de-meaning them. The statistics reported in Table 2 are computed using these de-meaned series.

percent non-SC/ST wage premium in the data, we consider the fit to be quite good.

The model allows for heterogeneity both within and across groups. To examine the fit of the model with regard to its predicted heterogeneity, we first fit a Pareto distribution to the years of schooling of agents separately for each caste in the NSS household survey data for 1983. We then do the same to the schooling outcomes in the model and compare the model with the data.

Table 2 reports the Pareto shape parameter estimated in the data and in the model for 1983. The model accurately generates thicker tails for the non-SC/ST education distribution relative to SC/STs. The quantitative fit of the shape parameter is very close for non-SC/STs but somewhat less so for SC/STs. We interpret this as evidence that the model performs well in matching the observed schooling heterogeneity in 1983. This is important since schooling heterogeneity is the key for the economic heterogeneity in the model.

Having described the fit of the model to the data moments in 1983, we now examine its dynamic predictions for caste gaps. Table 2 also gives the labor and wage gaps across castes in the model and the data in 2012. The main takeaway from the Table is in the last row. In the data, the wage gap between non-SC/STs and SC/STs declined by 0.15/1.45 or 10.3 percent between 1983 and 2012. The corresponding reduction generated by the model is 7.5 percent. Thus, the baseline model can explain 72 percent of the observed decline in the percentage wage gap.

Underneath the success in reproducing the overall caste wage gap dynamics, the model also has qualitative and quantitative success in generating the observed dynamics of the caste gaps in both sectoral wages and employment shares. Thus, the agricultural wage gap marginally increased during 1983-2012 while the services wage gap decreased, both in the data and in the model. Correspondingly, the model reproduces the relatively unchanged agricultural labor share gap as well as the very sharp decline in the services labor share gap in the data. This last feature is particularly important since, as we showed in the decomposition exercise, the size of the change in the caste labor share gap in services was the largest amongst all the sectoral gaps.

Where the model does not perform well is in matching the dynamics of the labor share and wage gaps in the manufacturing sector. In the data, the manufacturing labor gap widened by 10 percent while the model generates a 50 percent increase. The model also predicts an unchanged manufacturing wage gap while there was a marginal decrease in this gap in the data.

A key feature of the data is that there was a switch between the relative rank orders of the labor share gaps between manufacturing and services. While services had the largest caste gap in labor shares in 1983, by 2012 it was manufacturing that had the largest caste labor share gap. The model successfully reproduces this switch.

Table 2 also reports the change in the Pareto shape parameter for the schooling distribution of the two castes. Clearly, the model correctly matches the thickening tails of the schooling distribution for both castes, though with slightly more quantitative precision for non-SC/STs. We view this as evidence that sectoral productivity growth can account for a large part of the changes in the distribution of schooling outcomes in India since 1983.

4.1 Relative versus Absolute Convergence

The focal point of our paper is the convergence in relative wages between non-SC/STs and SC/STs. The focus on relative convergence is in keeping with the approach in the literature on inter-group inequalities which typically examines relative gaps between the groups of interest. Our focus on relative gaps is also consistent with the growth literature which looks at the relative income gaps across countries.

There is however, a parallel concern amongst some social scientists and policymakers about absolute inequality. Indeed, this is the reason why researchers sometimes use measures like the absolute Gini coefficient. Our model clearly has predictions for absolute wage convergence between castes. How do these predictions compare with the evidence on the behavior of absolute wage gaps between non-SC/STs and SC/STs during the period 1983-2012? Table 3 shows the change in the relative and absolute caste wage gaps in the data and in the model. We measure the relative caste wage gap at date t as w_{nt}/w_{st} and absolute caste wage gaps as $w_{nt} - w_{st}$. The table reports the percentage change between 1983 and 2012 in these two measures.

Table 3: Absolute versus Relative Convergence

	Change: 1983-2012	
Variable	Data	Model
Relative wage gap	-10.5	-7.5
Absolute wage gap	71.3	96.0

Note: The table reports changes in the relative and absolute wage gaps between non-SC/STs and SC/STs during 1983-2012

As the table shows, the 10.5 percent decrease in the relative caste wage gap during 1983-2012 in the data was accompanied by a 71.3 percent increase in the absolute wage gap between the two groups during the period. Reassuringly, the model reproduces this feature of the data as well by predicting a 96 percent rise in the absolute wage gap.

We view the results in Table 3 as independent evidence in support of the model since it was not calibrated to target the absolute gaps either in 1983 or in 2012.

4.2 Sectoral prices and quantities

An independent test of the model is how well it fits the aggregate facts on prices and quantities. Table 4 shows the percent change in sectoral prices and quantities in the data and their model counterparts during the period 1983-2012.

Table 4: Sectoral Prices and Quantities

		Percent change 1983-2012	
VARIABLE	Notation	Data	Model
Relative price Agri	p^a	+20.6%	+21.3%
Relative price Manuf	p^m	−8.5%	+21.0%
Relative price Serv	p^h	−6.2%	−26.0%
Output Share Agri	y^a	−75.0%	−19.6%
Output Share Manuf	y^m	+28.6%	−20.0%
Output Share Serv	y^h	+126.7%	+30.8%

Notes: The table reports the percent changes of sectoral prices and quantities in the data and the model.

Two features of the results in Table 4 are noteworthy. First, the model does well in matching the dynamics of the relative prices and quantities of agriculture and services. The predicted dynamics of the agricultural relative price is particularly important in this context. As the Table shows, the relative price of agriculture actually rose during 1983-2012 in India. The model matches this fact. We view this as a particular strength of the model since standard models of structural transformation which generate a declining share of agriculture over time have difficulty in simultaneously generating a rising agricultural relative price.[17]

Second, the model encounters difficulties in reproducing the dynamics of the manufacturing sector, both in quantities and prices. It predicts an increase in the relative price of manufacturing and a decrease in its output share. Both are counterfactual. This aspect of the model is similar to

[17] Standard models of structural transformation based on non-homothetic demand for the agricultural good predict that the relative price of agriculture declines in response to productivity growth since its demand rises less than proportionately with income. Models that generate structural transformation through inelastic elasticity of substitution across sectors predict that resources flow towards the slower growing non-agricultural sectors as their relative prices rise (see Ngai and Pissarides (2007)). But this is counterfactual in the Indian data during 1983-2012 when agriculture was the slowest growing sector.

its relative underperformance in matching the dynamics of the caste gaps in manufacturing.

5 Mechanism Underlying Convergence

The results above show that the model generates convergence in response to sectoral productivity shocks. What is the mechanism that generates this convergence? We investigate this issue by focusing on the two key margins that determine caste identities in the model. Recall that castes differ in the cost of schooling and the costs of accessing sectoral labor markets. These two costs induce a caste-specific sorting of agents into schooling and sectors which generates caste gaps in sectoral wages and employment. Changes in the caste wage gaps then are the result of differential changes in these caste specific schooling and labor market access costs which alter the schooling and sectoral choices by the two castes.

The two important cost parameters are the schooling cost λ and the entry cost parameter ϕ. Both of these are denominated in terms of the final good, and are *constant.* Hence, growth reduces the *real* costs of access to schooling and sectoral labor markets. The decline in these costs change the schooling and sectoral employment decisions of agents. Consequently, if these costs change at different rates for the two castes, then the sectoral caste gaps in employment and wages would change since the two groups would respond differently in their schooling and employment decisions.

We examine the individual importance of the schooling and labor market frictions by conducting two experiments. First, we scale the entry cost scaling parameter ϕ by the common growth factor. So, in this experiment, schooling costs become smaller due to growth but sectoral entry costs remain invariant. Second, we scale the schooling cost parameters $\lambda_j, j = n, s$ by the common growth factor while leaving the entry cost parameter unscaled. Hence, in this case, the entry costs fall with growth but schooling costs remain invariant.[18] The results are reported in Table 5.

The column "Scale entry cost" in Table 5 shows the percent changes in the predicted caste employment and wage gaps in response to the measured productivity growth when entry costs are scaled but schooling cost remain unscaled. Correspondingly, the column "Scale schooling cost" shows the changes in the various caste gaps in response to productivity growth when schooling costs are scaled but sectoral entry cost are not. Relative to the baseline case, the results show that the fall in real schooling costs due to growth is key for generating the convergence in the overall caste wage gap. Indeed, the convergence in the overall mean wage gap is even larger in this case.

[18]In these experiments we scale the relevant costs using the common growth factor $\frac{A_{2012}^{\theta}M_{2012}^{\eta}H_{2012}^{1-\theta-\eta}}{A_{1983}^{\theta}M_{1983}^{\eta}H_{1983}^{1-\theta-\eta}}$.

Table 5: Schooling and Sectoral Re-sorting

			Scaling costs	
Variable	Data	Baseline	Scale entry cost	Scale schooling cost
Δs^a	−1.25%	0.00%	0.00%	0.00%
Δs^m	9.79%	50.35%	230.07%	36.36%
Δs^h	−24.84%	−17.50%	−51.88%	5.00%
Δw^a	3.85%	0.96%	0.00%	0.00%
Δw^m	−5.00%	0.00%	10.83%	−0.01%
Δw^h	−8.28%	−20.00%	−27.08%	0.00%
Δw	−10.34%	−7.46%	−24.63%	5.22%

On the other hand, column "Scaling entry cost" shows that having only entry costs decline while leaving schooling costs unchanged would actually induce an increase in the wage gap.

To understand these results, recall from equations 3.26 and 3.27 that the key determinants of the caste gaps are the ratios of ability thresholds which, in turn, are dependent on the relative costs of schooling and sectoral entry of the two castes. The calibrated schooling costs are proportionately greater for SC/STs in 1983. Hence, growth reduces schooling costs relatively faster for SC/STs. On the other hand, the calibrated sectoral entry costs are proportionately greater for non-SC/STs in 1983. Hence, growth reduces entry costs relatively faster for non-SC/STs.

Scaling entry costs alone prevents growth from reducing the relatively higher entry costs of non-SC/STs while the relatively higher schooling cost for SC/STs continues to decline with growth. Since SC/STs benefit more from this, the predicted wage convergence is greater in this case. On the other hand, scaling schooling costs alone while leaving entry costs unscaled switches the benefits of growth disproportionately towards non-SC/STS. Hence, the wage gap widens in this case.[19]

These results indicate that the key mechanism driving the caste wage convergence in the model is the falling real costs of schooling in response to the productivity shocks.

6 Evidence on Schooling Costs

In order for growth to induce caste wage convergence, the model needs two features: (a) real costs of schooling to fall with growth; and (b) schooling costs to decline relatively faster for SC/STS. We now provide independent pieces of evidence in support of these features of the model.

[19] Scaling both costs leaves all caste gaps unchanged since neither threshold changes.

6.1 Schooling costs and growth

Do the real costs of schooling increase or decrease as economies grow? Banerjee and Duflo (2005) attempted an answer to this question by examining the relationship between average teacher salaries and per capita incomes across countries. Using a cross-country panel, they found that a one percent increase in per capita GDP was associated with a rise in average teacher salaries by significantly less than one percent. Hence, the ratio of teacher salaries to per capita GDP must fall as per capita GDP rises. Given that teacher salaries tend to account for 80-90 percent of the cost of providing education, they concluded that the real cost of schooling declines with growth.

We apply the approach of Banerjee and Duflo (2005) to India by using teacher salary data from various rounds of the NSS. The NSS reports teachers under seven different occupation codes which we detail in the Appendix. We compute the average teacher salary across all the teaching occupations for each state in India for each round of the survey. We then regress the log of the average state teacher salary on log per capita state domestic product (SDP) using a panel of India states from 1983 to 2011-12. The estimated equation after controlling for state fixed effects is:[20]

$$\ln(Teacher\_salary) = 2.886^{***} + 0.216^{**}\ln(SDP\_pc), \quad R^2 = 0.22, \; N = 155, \; FE : Yes \quad (6.28)$$

The coefficient on log per capita SDP is significantly less than one indicating teacher salaries rise less than proportionately with per capita incomes. This result provides direct evidence for decreasing real costs of schooling (schooling costs in terms of final output) in India, which is the key mechanism driving convergence in the model. It also corroborates the finding in the cross-country data obtained by Banerjee and Duflo (2005).

6.2 Relative schooling costs of SC/STS

The second feature of the model that generates convergence is a faster reduction of schooling costs for SC/STs relative to non-SC/STs. Is there any evidence for this? To answer this question, we use Census data from India for 1991 and 2011 to examine the distribution of schools across towns and villages in India.[21] Our data comes from the SHRUG open data platform made available by the Development Data lab. Details about the data can be found in Asher et al. (2021).

[20] The wage and SDP data are in constant 2004-05 rupees. *** indicates significance at the 0.1 percent level and ** indicates significance of the estimate at the 1 percent level.

[21] Digitized census data for India are available from 1991 onwards. This precludes the evaluation of school provisioning from 1981, which would have been closer to our household survey data start date of 1983.

We examine the differences between non-SC/ST, SC and ST dominated areas in the provisioning of schools during these years. Note that we separate SCs and STs in these exercises to provide greater clarity to the patterns.

We are especially interested in two questions: (a) were there fewer schools in SC/ST dominated villages and towns relative to non-SC/ST dominated area? (b) did school provisioning increase faster during 1991-2011 in SC/ST dominated areas relative to non-SC/ST areas? If the answers to these two questions are affirmative then it provides indicative evidence that schooling costs were indeed higher for SC/STs but also declined at a faster rate than the schooling costs for non-SC/STs.

Table 6 reports the key statistics on school provisioning. We follow Bailwal and Paul (2021) and define a village or town to be dominated by caste k if the majority of the population in the village or town belongs to caste k where $k = Non - SC/ST, SC, ST$.

Table 6: Provisioning of Schools

	1991			2011		
Area Dominance:	SC	ST	non-SC/ST	SC	ST	non-SC/ST
	Probability of having school in the village or town					
Primary	0.56	0.55	0.71	0.76	0.83	0.84
Middle	0.09	0.11	0.24	0.31	0.33	0.49
Secondary	0.04	0.04	0.11	0.11	0.10	0.21
	Fraction of people having school in the village or town					
Primary	90.1%	84.1%	91.4%	95.5%	96.1%	96.1%
Middle	45.9%	38.2%	52.9%	71.1%	59.6%	72.4%
Secondary	27.8%	22.6%	33.8%	41.9%	32.6%	46.7%
Obs	36,243	53,446	306,971	50,037	110,011	423,067

There are two takeaways from Table 6. First, the top panel of the Table shows that in 1991 Non-SC/ST dominated geographic areas had a higher probability of having schools of all types. School provisioning in SC and ST villages and towns has improved over time but the gap with Non-SC/ST areas still remained in 2011. [22]

Second, the bottom panel of the Table reports the fraction of the different groups that live in areas that provide the various kinds of schools. As in the top panel, the results show that relative to non-SC/STs, a smaller fraction SCs and STs live in areas which have schools for all three categories of schools. Importantly, the gaps were much smaller relative to 1991 indicating a faster increase in

[22] We estimate these probabilities by running logit regressions of a binary (1,0) variable indicating availability of school of type $j = Primary, Middle, Secondary$ in the town or village on a constant and dummy variables for SC and ST domination of the area.

school availability for SCs and STs as compared to non-SC/STs. [23]

We view these results as indicating that: (a) relative to non-SC/STs, schooling costs were greater for SC/STs in 1991; and (b) schooling costs declined relatively faster for SC/STs during 1991-2011. We interpret these findings as being supportive evidence for the schooling cost calibration for 1983 as well as their faster decline for SC/STs over time.[24]

An alternative interpretation of the schooling data above is that the faster increase in school provisioning in SC/ST dominated geographies after 1991 was due to public policy measures. Indeed, there were quite a few public policy initiatives since the 1990s that could have had this effect. The 73rd Amendment (1992) of the constitution provided for reservations for women and SC/STs in local governance. This could have improved local provisioning for education and resulted in differential outcomes in predominantly SC/ST areas. The District Primary Education Program (1993) and Sarva Shiksha Abhiyan (2000) could also have decreased the relative costs for SC/STs.

There is, however, considerable evidence that the spread of schooling after the 1990s was driven by a rapid expansion of private schools. Thus, Muralidharan and Kremer (2008) conducted a nationally representative survey in 2003 to document that: (a) 28 percent of rural Indians had access to a private school in their village; (b) most of the private schools were founded in the five years preceding 2003; (c) 40 percent of private school enrolment was in these newly founded schools; (d) the presence of private schools was *negatively* correlated with state and district per capita GDPs; and (e) the presence of a private school in a village was positively correlated with teacher absence rates in government schools in the village.

The results in Muralidharan and Kremer (2008) indicate that the growth of private schools was mainly a response to the absence of functioning public schools. We interpret this as evidence suggesting that there were important factors unrelated to public policy driving the expansion of school provisioning in India post-1991.[25]

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[23] Note that this measure is *not tied to whether a village or town is SC/ST dominated or not*. Instead, it directly measures the share of people that have school access where they live.

[24] In related work, Bailwal and Paul (2021) examine the distance to the nearest school from villages in India in 2001 and 2011 and find that (a) the distances to the nearest primary and middle schools are increasing in the village's SC and ST population shares; and (b) the positive correlation between distance to the nearest primary and middle schools and the SC/ST population share of the village declined between 2001 and 2011. While their sample period is different from our paper, nevertheless their finding (a) corroborates our calibration estimate of higher costs of schooling for SC/STs while their finding (b) provides support for a faster decrease in the cost of schooling for SC/STs during the sample.

[25] One could argue that post-1990s regulatory reform may have facilitated the growth of private schools. However, the major thrust of the criticisms of education policy changes during this period has been that they made the operation of private schools more onerous rather than less. For an in-depth review of the role of private schools in India, see Muralidharan (2019).

7 Misallocations and Productivity

A key aspect of our model is that labor productivity responds to both exogenous and endogenous factors. The endogenous response arises anytime agents change their schooling and sectoral employment decisions in response to exogenous shocks. This re-sorting changes the human capital of workers as well as the sectoral distribution of the human capital, both of which affect the sectoral and aggregate levels of labor productivity. Put differently, productivity affects talent misallocation but misallocation itself affects productivity in the model. How big is this latter effect?

As we saw above, worker re-sorting in the model occurs due to changing costs of schooling and sectoral employment which also change the talent misallocations and caste gaps. We evaluate the quantitative importance of the decreasing caste misallocation for labor productivity by comparing sectoral labor productivity growth rates under three scenarios: (i) sectoral entry costs are scaled to aggregate growth; (ii) schooling costs are scaled to aggregate growth; and (iii) both entry costs and schooling costs are scaled to growth. In each experiment, we hit the model with the imputed exogenous productivity growth rates in the baseline case.

Table 7 reports the sectoral productivity growths under the three scenarios as well as the numbers in the baseline case. Comparing the column "Scale both" with the last column shows that the overall and exogenous sectoral productivity growth rates become identical when both costs are scaled. In this case there is no change in misallocations as the relative real costs of schooling and sectoral employment remain unchanged for the two castes. Hence, the difference between the exogenous sectoral productivity growth and the overall sectoral productivity growth in the baseline case (column "Baseline") is due to changing misallocations.

Table 7: Changing Misallocations and Productivity Growth

		Scaling costs				
Variable	Data	Baseline	Scale entry	Scale educ	Scale Both	Exogenous Growth
$\frac{Ey_{11}^a}{Ey_{83}^a}$	2.07	2.07	2.10	1.00	1.14	1.14
$\frac{Ey_{11}^m}{Ey_{83}^m}$	3.40	3.40	3.78	1.67	2.14	2.14
$\frac{Ey_{11}^h}{Ey_{83}^h}$	2.70	2.70	2.72	1.71	2.41	2.41

The numbers in Table 7 imply that declining misallocations due to endogenous education and sectoral sorting account for 45 percent, 37 percent and 11 percent of overall labor productivity growth in Agriculture, Manufacturing and Services, respectively. If one weights these numbers by the sectoral share parameters θ, η and $1 - \theta - \eta$, declining talent misallocation accounts for 39 percent of the labor productivity growth in India during 1983-2012.

8 Welfare Costs of Caste Distortions

The model that we have outlined has two sources of differences across castes: the costs of schooling and the costs of entry into sectoral labor markets. How expensive are these distortions? How much would SC/ST welfare rise if these distortions were removed? Would non-SC/STs gain as well? What would be the aggregate welfare gains?

In order to interpret the differences across castes in schooling and sectoral entry costs as distortions, we now provide a tax representation of these costs. Specifically, we define:

$$\lambda_s = \lambda_n + \tau_\lambda$$
$$\gamma_s^k = \gamma_n^k + \tau_\gamma^k, \quad k = m, h$$

where τ_λ is the tax on schooling and $\tau_\gamma^k, k = m, h$ is the tax on sectoral entry costs borne by SC/ST agents. Note that since $\gamma_s^k < \gamma_n^k,\ k = m, h$ under our calibration in in Table 1, $\tau_\gamma^k < 0,\ k = m, h$, i.e., affirmative action will act as a subsidy for SC/STs in accessing sectoral labor markets.

Using T_i to denote per capita public expenditure, the government's budget constraint is

$$L\left[s\int_{\underline{a}}^{\bar{a}} T_i dG(a_i) + n\int_{\underline{a}}^{\bar{a}} T_i dG(a_i)\right] = L\left[s\int_{\underline{a}}^{\bar{a}} \tau_\lambda q_{i,s}^* dG(a_i)\right] + L\left[s\int_{\hat{a}_s^m}^{\tilde{a}_s^h} \tau_\gamma^m \phi dG(a_i) + s\int_{\tilde{a}_s^h}^{\bar{a}} \tau_\gamma^h \phi dG(a_i)\right] \tag{8.29}$$

where $q_{i,s}^*$ stands for the optimal choices of schooling given by equations 3.7-3.9.

This formulation of the cost differences as tax distortions leaves unchanged the production details of the model since we retain the same calibrated $\lambda_n, \lambda_n, \gamma_n^k, \gamma_s^k$ as in Table 1. The consumption side of the model however does get affected by this reformulation since taxes could be either rebated to the public or consumed by the government. We shall examine both possibilities below.

To assess the welfare costs of caste distortions, we compare aggregate outcomes under the baseline case with two sets of counterfactual experiments: (a) equal sectoral entry costs for the two castes; and (b) equal schooling and sectoral entry costs. We conduct this comparison both with and without tax rebates. Note that since the two castes draw their ability endowments from the same distribution, equalizing both caste distortions would eliminate all caste gaps.

Table 8: Welfare Costs of Caste Distortions Under No Rebate

	1983			2012		
Variable	Baseline	γ's equal	all equal	Baseline	γ's equal	all equal
C_s	101.29	98.22	160.1	226.78	223.29	349.0
C_n	160.00	160.27	160.1	346.45	349.02	349.0
C	145.32	144.76	160.1	316.53	317.59	349.0
Y_a	134.04	133.72	146.3	287.97	288.91	310.6
Y_m	220.68	224.42	241.1	485.35	479.88	515.3
Y_h	293.97	288.55	325.6	1025.8	1035.57	1127.0
Y_f	195.87	194.73	218.13	510.71	512.50	553.6

Notes: The table reports average consumption of each caste as well as per capita outputs of the sectoral and final goods under various parameter configurations for schooling and sectoral entry costs when taxes are not rebated to the public.

Table 8 reports the results for the case when taxes are not rebated. The last column of the Table ("all equal") in the left panel (1983) shows that equalizing all costs equalizes average consumption for both castes since they both draw from the same ability distribution. This translates into an increase in average consumption for SC/STs by 58.8 percent in 1983 and 53.9 percent in 2012. Interestingly, it also marginally raises the average consumption of non-SC/STs in both years. This occurs due to the rise in aggregate output that is induced by the removal of caste distortions.

Aggregate output, Y_f, rises by 11.4 percent in 1983. This is the static gain from removing caste distortions. The corresponding output gain in 2012 is 8.4 percent. The increase in average per capita consumption, C, from removing all caste distortions in this economy is 10.2 percent in 1983 and 10.3 percent in 2012.

How do these estimates change when the caste taxes are rebated back to the public in the form of lump-sum transfers? Table 9 reports the results for average consumption in this case. Since the production side of the economy is unaffected by whether taxes are rebated or not, the output numbers in this case are identical to those in Table 8 above.

As one might expect, the tax rebate raises the average consumption of both castes in the distorted baseline economy relative to the no-rebate case. Outcomes when all distortions are removed however remain identical to those in Table 8. Consequently, the welfare gains for SC/STs from removing all distortions are now smaller in both years. The average per capita consumption gains

Table 9: Welfare Costs of Caste Distortions Under Lump-Sum Rebates

	1983			2012		
Variable	Baseline	γ's equal	all equal	Baseline	γ's equal	all equal
C_s	110.94	107.48	160.1	250.33	245.39	349.0
C_n	169.65	169.53	160.1	370.67	371.12	349.0
C	154.97	154.01	160.1	340.58	339.69	349.0

Notes: The table reports average consumption of each caste as well as per capita outputs of the sectoral and final goods under various parameter configurations for schooling and sectoral entry costs when taxes are fully rebated to the public.

for SC/STs is 44.3 percent in 1983 and 39.4 percent in 2012.

The interesting feature of the full rebate case is that removal of all distortions now does hurt non-SC/STs. Since non-SC/STs receive net positive transfers from SC/STs through the tax rebates under the distorted economy, the removal of all taxes reduces their net income. This effect is strong enough for removal of distortions to cause a reduction in the average consumption of non-SC/STs.

The main takeaway from these results is that there are significant welfare gains from removing caste distortions. These gains are very high for SC/STs. Strikingly, in the realistic case of no tax rebates, reforms that remove all caste distortions also raise the welfare of non-SC/STs indicating that the reforms are Pareto improvements.

9 Structural Transformation and Affirmative Action

Our approach has two features that require additional discussion. The first is the role of the structural transformation: how important is that for the predicted wage convergence? The second is the importance of affirmative action policies for our results.

9.1 Caste Gaps and Structural Transformation

The baseline model has three mechanisms that induce structural transformation: (a) non-homothetic production technology; (b) differential sectoral productivity growth; and (c) unscaled schooling and sectoral entry costs. The first two are standard in many models of structural transformation. The last one is more specific to our model.

We examine the implications of structural transformation for the caste gaps by conducting a quantitative experiment where we remove all the ingredients in the model that induce structural transformation. Thus, we compare the baseline model to one where we impose a common sectoral productivity growth (set at the aggregate growth rate), set $\bar{y} = 0$, and scale all the costs by making both λ_j and $\gamma_j^k,\ k = m, h;\ j = n, s$ proportional to the growth rate of aggregate output.

Table 10: Role of Structural Transformation

Changes under common growth and scaled costs 1983-2012					
	Caste Gaps			Aggregate Sectoral Shares	
Variable	Baseline	$\bar{y} = 0$	Variable	Baseline	$\bar{y} = 0$
Δs^a	0.15%	0.00%	SL^a	$0.68 - 0.71$	$0.68 - 0.68$
Δs^m	50.23%	0.00%	SL^m	$0.19 - 0.12$	$0.19 - 0.19$
Δs^h	−17.41%	0.00%	SL^h	$0.13 - 0.17$	$0.13 - 0.13$
Δw^a	0.26%	0.00%	SY^a	$0.46 - 0.37$	$0.46 - 0.46$
Δw^m	0.38%	0.00%	SY^m	$0.15 - 0.12$	$0.15 - 0.15$
Δw^h	−19.96%	0.00%	SY^h	$0.39 - 0.51$	$0.39 - 0.39$
Δw	−7.53%	0.00%			

Notes: The left panel of the table gives changes in the caste gaps in sectoral employment and wages between 1983 and 2012 in the baseline case and in the case when $\bar{y} = 0$, common sectoral growth rate, and scaled schooling and sectoral entry costs. The right panel gives the corresponding changes under the baseline case and in the case with $\bar{y} = 0$, common sectoral growth rates and scaled costs. SL^k denotes the employment share of sector $k = a, m, h$. SY^k denotes the output share of sector $k = a, m, h$.

The main takeaway from the numbers in Table 10 under the columns for $\bar{y} = 0$ is that without the conditions that generate structural transformation, productivity changes have no impact on the caste employment and wage gaps. Intuitively, when all sectors grow at a common rate, ability thresholds for the two castes are unaffected by productivity growth since the rewards from switching sectors change at the same rate as the costs of schooling and accessing sectors.

This result is indicative of the importance of structural transformation in the Indian economy during 1983-2012 for understanding the dynamic evolution of the caste gaps during this period.

9.2 Affirmative Action Policies

The Indian constitution mandates reservations of seats in public institutions of tertiary education, in public sector employment and in political representation for SC/STs. How important were these reservations for the observed caste convergence between 1983 and 2012?

Recall that our calibration of the model for 1983 dictated lower fixed costs of accessing manufacturing and service sector employment for SC/STs. We view these lower sectoral entry costs of

SC/STs as the proxy for reservations in the model. To examine the importance of reservations, we conducting three counterfactual simulations: (a) $\frac{\gamma_n^m}{\gamma_s^m} = 1$; (b) $\frac{\gamma_n^h}{\gamma_s^h} = 1$; and (c) $\frac{\gamma_n^m}{\gamma_s^m} = \frac{\gamma_n^h}{\gamma_s^h} = 1$. In these experiments we leave γ_n^m and γ_n^h at their baseline levels. In other words, we raise the fixed cost component of sectoral entry costs for SC/STs to non-SC/STs levels in each sector thereby *eliminating the advantage* of reservations for SC/STs. All other baseline parameters remain unchanged.

Table 11: Role of Affirmative Action

	1983					2012				
Variable	Data	Baseline	γ^m	γ^h	both	Data	Baseline	γ^m	γ^h	both
Δs^a	0.80	0.85	0.84	0.85	0.84	0.79	0.85	0.84	0.85	0.84
Δs^m	1.43	1.43	1.54	0.79	0.84	1.57	2.15	2.54	0.77	0.84
Δs^h	1.61	1.60	1.58	93.37	82.78	1.21	1.33	1.31	3.90	3.84
Δw^a	1.04	1.04	1.01	1.04	1.01	1.08	1.05	1.01	1.05	1.01
Δw^m	1.20	1.20	1.18	1.01	1.00	1.14	1.20	1.18	1.02	1.00
Δw^h	1.45	1.44	1.45	1.26	1.26	1.33	1.16	1.16	1.02	1.02
Δw	1.45	1.34	1.31	1.62	1.58	1.30	1.24	1.22	1.33	1.31

Notes: For $j = a, m, h$, Δs^j is the ratio of the fraction of all non-SC/STs working in sector j to the fraction of all SC/STs working in sector j; Δw^j is the ratio of the mean non-SC/ST to mean SC/ST wage in sector j; Δw is the ratio of the mean non-SC/ST to mean SC/ST wage.

The left panel of Table 11 shows the effects of equalizing sectoral entry costs in 1983 while the right panel shows the corresponding effects in 2012. Comparing the "both" and "Baseline" columns in the Table reveals that when both sectoral entry costs are equalized, the overall wage gap in 1983 rises to 1.58 from the baseline level of 1.34 while for 2012 the gap rises to 1.31 from the baseline of 1.24. Clearly, reservations reduced the level of the wage gap at all dates. However, since the *level* of the gap rises at both dates, removing reservations has a muted effect on its *dynamics*.

We view these results as indicating that affirmative action policies may have affected the level of the wage gaps but not their dynamics during 1983-2012.

10 Conclusion

The paper has examined the role of growth in accounting for the observed convergence in the education, occupation choices and wages of scheduled castes and tribes (SC/STs) in India toward the levels of non-SC/STs during 1983-2012.

We formalized a multi-sector, heterogenous agent model where all individuals draw their innate ability from the same ability distribution but their costs of acquiring schooling and accessing sectoral labor markets depend on their caste. We examined the dynamic effects of exogenous productivity growth on caste gaps in this environment with caste-based talent misallocations. Our quantitative experiments on the model suggest that exogenous sectoral productivity growth can account for 72 percent of the observed wage convergence between SC/STs and non-SC/STs during 1983-2012.

The main mechanism driving the caste convergence in the model is SC/STs increasing their relative education levels in response to decreasing real costs of schooling. We provide independent evidence in support of this mechanism using India-wide data on teacher salaries and school provisioning. We show that: (a) teacher salaries rise less than proportionately with per capita income so that cost of schooling declines with growth; and (b) villages and towns dominated by SC/STs enjoyed a faster increase in provisioning of schools between 1991 and 2011.

Equilibrium labor productivity in the model depends on exogenous productivity as well as the sorting of workers in education and sectoral employment. We find that the exogenous productivity growth induced a re-sorting of workers that reduced the caste-based talent misallocation. The resultant endogenous increase in labor productivity accounted for 44 percent, 37 percent and 11 percent of the overall sectoral labor productivity growth in Agriculture, Manufacturing and Services during 1983-2012. Clearly, the productivity payoffs of declining misallocations were large.

The model estimates the welfare costs of caste-based misallocations to be greater than 10 percent of per capita consumption. Clearly, castes represent a significant distortion. However, growth during 1983-2012 appears to have succeeded in breaking down millenia of the socio-economic disparities induced by these caste distortions. Our results suggest that growth focused policies may be more potent than group-based redistribution policies in reducing inter-group inequalities.

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11 Appendix

11.1 Data

Our primary data source is the National Sample Survey (NSS) employment-unemployment household surveys from 1983 to 2011-12. We consider individuals between the ages 16-65 belonging to male-headed households who were not enrolled full time in any educational degree or diploma. The sample is restricted to those individuals who provided their 4-digit industry of employment information as well as their education information.[26]

Our focus is on full-time working individuals who are defined as those that worked at least 2.5 days per week. This selection leaves us with a working sample of around 165,000-182,000 individuals, depending on the survey round. The wage data is more limited. This is primarily due

[26] We also consider a narrower sample in which we restrict the sample to only males and find that our results remain robust.

to the prevalence of self-employed individuals in rural India who do not report wage income. This limits the sub-sample with wage data to about 48,000 individuals on average across rounds.

In the text we group the reported industry codes into three broad industry categories: Ind 1 refers to Agriculture, Hunting, Forestry and Fishing; Ind 2 collects Manufacturing and Mining and Quarrying; Ind 3 refers to all Service industries. These groupings are detailed in Table 12.

Table 12: Industry categories

Industry code	Industry description	Group
A	Agriculture, Hunting and Forestry	Ind 1
B	Fishing	Ind 1
C	Mining and Quarrying	Ind 2
D	Manufacturing	Ind 2
E	Electricity, Gas and Water Supply	Ind 3
F	Construction	Ind 3
G	Wholesale and Retail Trade; Repair of Motor Vehicles, motorcycles and personal and household goods	Ind 3
H	Hotels and Restaurants	Ind 3
I	Transport, Storage and Communications	Ind 3
J	Financial Intermediation	Ind 3
K	Real Estate, Renting and Business Activities	Ind 3
L	Public Administration and Defence; Compulsory Social Security	Ind 3
M	Education	Ind 3
N	Health and Social Work	Ind 3
O	Other Community, Social and Personal Service Activities	Ind 3
P	Private Households with Employed Persons	Ind 3
Q	Extra Territorial Organizations and Bodies	Ind 3

11.2 Structural transformation

Figure 6 shows that 1983-2012 witnessed a gradual contraction in the agricultural sector along with an expansion of the service sector.

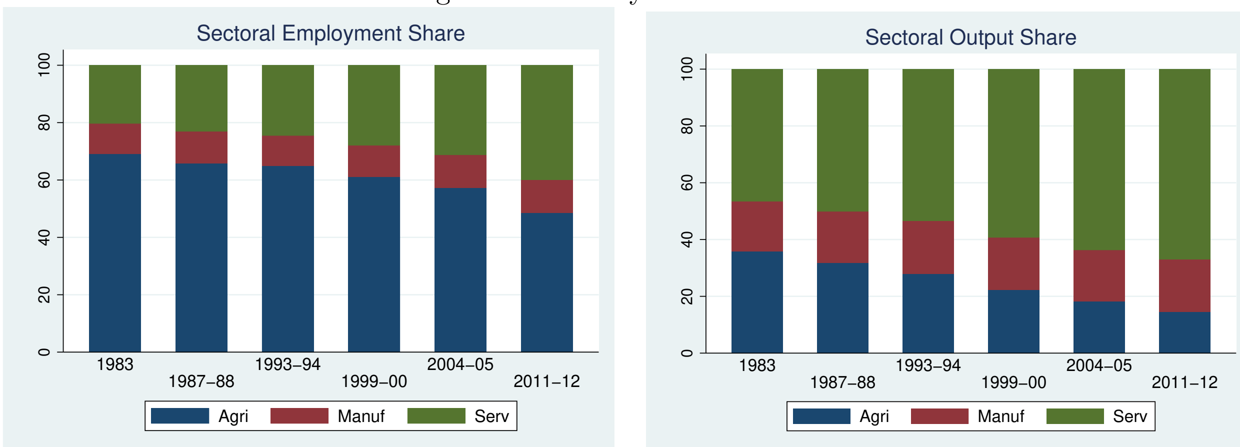
11.3 Teacher occupation data

The NSSO data classifies the teaching profession under 3 primary NCO codes and 4 teaching associate professional codes. The details are provided in the table below.

Table 13: Teacher occupation categories

NCO Code	Description
231	College University and High School education teaching professionals
232	Secondary education teaching professional
233	Other teaching professional
331	Middle and primary teaching associate professional
332	Pre-primary teaching associate professional
333	Special education teaching associate professional
334	Other teaching associate professional

Figure 6: Industry distribution



(a) (b)

Notes: Panel (a) gives the industrial distribution of workers for different NSS rounds. Panel (b) presents distribution of output (measured in constant 1980-81 prices) across three industry categories.

11.4 Proofs of Lemmas 3.1 and 3.2

To ease notation, throughout this section we will use the definition:

$$\Psi_j \equiv (1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}}$$

Lemma 3.1 *All individuals $i \in$ caste $j = n, s$ with ability a_{ij} prefer employment in sector-m to employment in sector-a if $a_{ij} \geq \hat{a}^m_j$; employment in sector-h to sector-a if $a_{ij} \geq \hat{a}^h_j$; and employment in sector-h to sector-m if $a_{ij} \geq \tilde{a}^h_j$.*

Proof. The agent will choose the sector that gives the highest c^k_{ij}. It is easy to see that the agent prefers sector a to m if and only if $c^a_{ij} \geq c^m_{ij}$. Similarly, she prefers a to h iff $c^a_{ij} \geq c^h_{ij}$ and m to h if and only if $c^m_{ij} \geq c^h_{ij}$ where c^a_{ij}, c^m_{ij} and c^h_{ij} are given by equations 3.10, 3.11 and 3.12, respectively. We can rewrite these three conditions and define:

$$z^m_j(a_{ij}) \equiv \frac{\phi\gamma^m_j}{a_{ij}^{\frac{1}{1-\chi}}} \geq \Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}} \quad (11.30)$$

$$z^h_j(a_{ij}) \equiv \frac{\phi\gamma^h_j}{a_{ij}^{\frac{1}{1-\chi}}} \geq \Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}} \quad (11.31)$$

$$z^h_j(a_{ij}) - z^m_j(a_{ij}) \equiv \frac{\phi(\gamma^h_j - \gamma^m_j)}{a_{ij}^{\frac{1}{1-\chi}}} \geq \Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}} \quad (11.32)$$

With $0 < \chi < 1$, $\phi, \gamma_j^k > 0$ and Assumption 2, it is obvious that $z_j^m(a_{ij})$, $z_j^h(a_{ij})$ and $z_j^h(a_{ij}) - z_j^m(a_{ij})$ are strictly decreasing in a_{ij}. Since $p^h H + \phi\alpha > p^m M + \phi\alpha > p^a A$ (Assumption 3), we have:

$$\begin{cases} c_{ij}^a \leq c_{ij}^m & \text{iff } a_{ij} \geq \hat{a}_j^m \\ c_{ij}^a \leq c_{ij}^h & \text{iff } a_{ij} \geq \hat{a}_j^h \\ c_{ij}^m \leq c_{ij}^h & \text{iff } a_{ij} \geq \tilde{a}_j^h \end{cases}$$

■

Lemma 3.2: *The rank order of the three ability thresholds are*

$$\tilde{a}_j^h < \hat{a}_j^h < \hat{a}_j^m \quad \text{if} \quad \hat{a}_j^h = min[\hat{a}_j^m, \hat{a}_j^h]$$
$$\tilde{a}_j^h > \hat{a}_j^h > \hat{a}_j^m \quad \text{if} \quad \hat{a}_j^h = max[\hat{a}_j^m, \hat{a}_j^h]$$

Proof. Consider first the case $\hat{a}_j^h < \hat{a}_j^m$. In this case, suppose $\tilde{a}_j^h > \hat{a}_j^h$. Using the definitions of $\hat{a}_j^h$ and $\tilde{a}_j^h$ from equations 3.14 and 3.15 above, $\tilde{a}_j^h > \hat{a}_j^h$ can be rewritten as

$$\left[\frac{\phi\gamma^h}{(1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}}\left\{(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - (p^a A)^{\frac{1}{1-\chi}}\right\}}\right]^{1-\chi} > \left[\frac{\phi\gamma^m}{(1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}}\left\{(p^m M + \phi\alpha)^{\frac{1}{1-\chi}} - (p^a A)^{\frac{1}{1-\chi}}\right\}}\right]^{1-\chi}$$

But this implies that $\hat{a}_j^h > \hat{a}_j^m$ which is a contradiction. Hence, if $\hat{a}_j^h < \hat{a}_j^m$ then $\tilde{a}_j^h < \hat{a}_j^h < \hat{a}_j^m$.

The other case $\hat{a}_j^h > \hat{e}_j^m$ but $\hat{a}_j^h > \tilde{a}_j^h$ leads to a contradiction by a similar logic. Hence, if $\hat{a}_j^h > \hat{a}_j^m$ then $\tilde{a}_j^h > \hat{a}_j^h > \hat{a}_j^m$. ■