Online Appendix for Convergence Across Castes

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1 Household's Problem

Given the final production function, an agent i belonging to caste j chooses optimal intermediate good to maximize:

$$\max_{y_{ij}^a, y_{ij}^m, y_{ij}^h} (y_{ij}^a - \bar{y})^{\theta} (y_{ij}^m)^{\eta} (y_{ij}^h)^{1-\theta-\eta} \quad s.t. \quad w_{ij} \geq p^a y_{ij}^a + p^m y_{ij}^m + p^h y_{ij}^h \tag{1}$$

Taking w_{ij} as given, solve for the FOCs:

$$p^a \eta (y_{ij}^a - \bar{y}) = \theta p^m y_{ij}^m \tag{2}$$

$$p^a (1 - \theta - \eta)(y_{ij}^a - \bar{y}) = \theta p^h y_{ij}^h \tag{3}$$

Using the above FOCs and the constraint we get the demand functions in main text:

$$p^a y_{ij}^a = p^a \bar{y} + \theta (w_{ij} - p^a \bar{y}) \tag{4}$$

$$p^m y_{ij}^m = \eta (w_{ij} - p_a \bar{y}) \tag{5}$$

$$p^h y_{ij}^h = (1 - \eta - \theta)(w_{ij} - p^a \bar{y}) \tag{6}$$

2 Education and Sectoral Choices

In the main text, the final good consumption contingent to sectoral choice $k = a, m, h$ is defined as:

$$c_{ij}^k = y_{ij}^k - \lambda_j q_{ij}^k - f_{ij}^k \tag{7}$$

where y_{ij}^k is the final good production of agent i when employed in sector k:

$$y_{ij}^k = w_{ij}^k - p^a \bar{y}$$

The wage w_{ij}^k will depend on the sector k:

$$w_{ij}^k = \begin{cases} p^a A e_{ij} = p^a A q_{ij}^{\chi} a_{ij} & k = a \\ p^m M e_{ij} = p^m M q_{ij}^{\chi} a_{ij} & k = m \\ p^h H e_{ij} = p^h H q_{ij}^{\chi} a_{ij} & k = h \end{cases} \tag{8}$$

Education Choice: Given the formulation above and the form of f_{ij}^k defined in Assumption 1, take F.O.C with respect to q_{ij} to get sectoral specific schooling:

$$q_{ij}^a = \left[\frac{\chi a_{ij} p^a A}{\lambda_j}\right]^{1/(1-\chi)} \tag{9}$$

$$q_{ij}^m = \left[\frac{\chi a_{ij} (p^m M + \phi\alpha)}{\lambda_j}\right]^{1/(1-\chi)} \tag{10}$$

$$q_{ij}^h = \left[\frac{\chi a_{ij} (p^h H + \phi\alpha)}{\lambda_j}\right]^{1/(1-\chi)} \tag{11}$$

Sectoral Choice: Plug (9)-(11) into (7) and get:

$$c_{ij}^a = (1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} (a_{ij} p^a A)^{\frac{1}{1-\chi}} \tag{12}$$

$$c_{ij}^m = (1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \{a_{ij}(p^m M + \phi\alpha)\}^{\frac{1}{1-\chi}} - \phi\gamma_j^m \tag{13}$$

$$c_{ij}^h = (1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}} \left\{a_{ij}\left(p^h H + \phi\alpha\right)\right\}^{\frac{1}{1-\chi}} - \phi\gamma_j^h \tag{14}$$

The agent will choose the sector that gives the highest c_{ij}^k. To ease some notations, we define:

$$\Psi_j = (1-\chi)\left(\frac{\chi}{\lambda_j}\right)^{\frac{\chi}{1-\chi}}$$

First it is easy to see that agent prefers sector a to m if and only if:

$$c_{ij}^a = \Psi_j (p^a A)^{\frac{1}{1-\chi}} a_{ij}^{\frac{1}{1-\chi}} - p^a \bar{y} \geq c_{ij}^m = \Psi_j (p^m M + \phi\alpha)^{\frac{1}{1-\chi}} a_{ij}^{\frac{1}{1-\chi}} - \phi\gamma_j^m - p^a \bar{y} \tag{15}$$

Similarly, she prefers a to h iff $c_{ij}^a \geq c_{ij}^h$ and m to h iff $c_{ij}^m \geq c_{ij}^h$. We can rewrite these three conditions and define:

$$z_j^m(a_{ij}) \equiv \frac{\phi\gamma_j^m}{a_{ij}^{\frac{1}{1-\chi}}} \geq \Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}} \tag{16}$$

$$z_j^h(a_{ij}) \equiv \frac{\phi\gamma_j^h}{a_{ij}^{\frac{1}{1-\chi}}} \geq \Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}} \tag{17}$$

$$z_j^h(a_{ij}) - z_j^m(a_{ij}) \equiv \frac{\phi(\gamma_j^h - \gamma_j^m)}{a_{ij}^{\frac{1}{1-\chi}}} \geq \Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}} \tag{18}$$

We then define the cut-off ability levels when equalities bind:

$$\hat{a}_j^m = \left[\frac{\phi\gamma_j^m}{\Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}}}\right]^{1-\chi} \tag{19}$$

$$\hat{a}_j^h = \left[\frac{\phi\gamma_j^h}{\Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}}}\right]^{1-\chi} \tag{20}$$

$$\tilde{a}_j^h = \left[\frac{\phi(\gamma_j^h - \gamma_j^m)}{\Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}}}\right]^{1-\chi} \tag{21}$$

The sectoral choices are then given by Proposition 4.1 in the main text.

3 Proofs

In this section we sketch the proofs of **Lemma 4.1** and **Lemma 4.2** in the main text.

3.1 Lemma 4.1

Lemma 1. *All individuals $i \in$ caste $j = n, s$ with ability a_{ij} prefer employment in sector-m to employment in sector-a if $a_{ij} \geq \hat{a}_j^m$; employment in sector-h to sector-a if $a_{ij} \geq \hat{a}_j^h$; and employment in sector-h to sector-m if $a_{ij} \geq \tilde{a}_j^h$.*

Proof. With $0 < \chi < 1$, $\phi, \gamma_j^k > 0$ and Assumption 2, it is obvious that $z_j^m(a_{ij})$, $z_j^h(a_{ij})$ and $z_j^h(a_{ij}) - z_j^m(a_{ij})$ defined by (16)-(18) are strictly decreasing in a_{ij}. From Assumption 3 that $p^h H + \phi\alpha > p^m M + \phi\alpha > p^a A$, we know that:

$$\begin{cases} c_{ij}^a \leq c_{ij}^m & \text{iff } a_{ij} \geq \hat{a}_j^m \\ c_{ij}^a \leq c_{ij}^h & \text{iff } a_{ij} \geq \hat{a}_j^h \\ c_{ij}^m \leq c_{ij}^h & \text{iff } a_{ij} \geq \tilde{a}_j^h \end{cases}$$

$\square$

3.2 Lemma 4.2

Lemma 2. *The rank order of the three ability thresholds are*

$$\tilde{a}_j^h < \hat{a}_j^h < \hat{a}_j^m \quad \textit{if} \;\; \hat{a}_j^h = min[\hat{a}_j^m, \hat{a}_j^h]$$
$$\tilde{a}_j^h > \hat{a}_j^h > \hat{a}_j^m \quad \textit{if} \;\; \hat{a}_j^h = max[\hat{a}_j^m, \hat{a}_j^h]$$

Proof. Without loss of generality, I'll show the rank order for the case $\hat{a}_j^h > \hat{a}_j^m$. The other case follows the same rationale. From equality of (18):

$$z_j^h(\tilde{a}_j^h) - z_j^m(\tilde{a}_j^h) = \Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}}$$

Then from the equalities of (16) and (17):

$$z_j^h(\hat{a}_j^h) - z_j^m(\hat{a}_j^m) = \Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}}$$

So we have:

$$z_j^h(\tilde{a}_j^h) - z_j^m(\tilde{a}_j^h) = z_j^h(\hat{a}_j^h) - z_j^m(\hat{a}_j^m)$$

Now suppose 1) $\hat{a}_j^m < \tilde{a}_j^h < \hat{a}_j^h$, because $z_j^m(a), z_j^h(a)$ and $z_j^h(a) - z_j^m(a)$ are all strictly decreasing functions:

$$z_j^h(\tilde{a}_j^h) > z_j^h(\hat{a}_j^h), \quad z_j^m(\tilde{a}_j^h) < z_j^m(\hat{a}_j^m) \quad \Rightarrow \quad z_j^h(\tilde{a}_j^h) - z_j^m(\tilde{a}_j^h) > z_j^h(\hat{a}_j^h) - z_j^m(\hat{a}_j^m)$$

This is a contradiction.

Now suppose 2) $\tilde{a}_j^h < \hat{a}_j^m < \hat{a}_j^h$. There exists $a \in (\hat{a}_j^m, \hat{a}_j^h)$ so:

$$\begin{aligned}
&z_j^h(a) > z_j^h(\hat{a}_j^h) = \Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}} \\
&z_j^m(a) < z_j^m(\hat{a}_j^m) = \Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^a A)^{\frac{1}{1-\chi}} \\
\Rightarrow &z_j^h(a) - z_j^m(a) > \Psi_j(p^h H + \phi\alpha)^{\frac{1}{1-\chi}} - \Psi_j(p^m M + \phi\alpha)^{\frac{1}{1-\chi}} = z_j^h(\tilde{a}_j^h) - z_j^m(\tilde{a}_j^h) \\
\Rightarrow &a < \tilde{a}_j^h
\end{aligned}$$

This is again a contradiction.

The case 1) and 2) lead to the ranking $\tilde{a}_j^h > \hat{a}_j^h > \hat{a}_j^m$ if $\hat{a}_j^h = \max[\hat{a}_j^m, \hat{a}_j^h]$. The other case follows the same rationale. $\square$